TRANSPORT COEFFICIENTS FOR LOW ENERGY ELECTRONS IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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Summary

Experimental results are given for the ratio W_x/W_z of transverse to longitudinal drift velocity for electron swarms in nitrogen moving in crossed electric and magnetic fields. The results, obtained by Huxley's method, cover the range $0\cdot04 < E/p < 8\cdot0$ V cm^{-1} torr^{-1} at 293°K. The apparatus and experimental procedures which have been developed permit accurate measurements to be made so that significant tests of the method have been possible over wide ranges of the experimental parameters. Information concerning the variation of the momentum transfer cross section with electron energy, and concerning the energy distribution function, can be obtained by comparing a quantity

$$W_M = (E/B)\,(W_x/W_z)$$

with the true drift velocity W. The results of this comparison are discussed in relation to recent theoretical analyses.

I. Introduction

Measurements of the deflection of electron swarms, drifting in steady d.c. electric fields, produced by a magnetic field at right angles to the electric field date back to the pioneering work of J. S. Townsend (Townsend and Tizard 1913). Notwithstanding the revived interest in the measurement of transport coefficients for low energy electrons drifting and diffusing through gases at pressures ranging from several torr to several hundred torr (McDaniel 1964, and references therein) little has been done to add to this early work of Townsend and its extension by his colleagues to a number of other gases (Healey and Reed 1941, and references therein; Townsend 1948).

The early experiments were designed specifically to measure drift velocities and the data determined from them were unique for many years. In fact, because of a fundamental limitation to time-of-flight methods,† the only available data for electron drift velocities at high values of E/p (ratio of electric field strength to gas pressure) are those derived from experiments of this type. However, as has long been realized (Townsend 1937), there are difficulties in deducing precise values of drift velocity from the measurements. When the method was first applied, Townsend deduced a simple relation between the angular deflection of the stream and the

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† At high values of E/p it is difficult, if not impossible, to obtain sufficiently well-defined electron groups in an apparatus of adequate length to enable the transit time of the groups to be measured with any precision. To do so requires values of pd (product of gas pressure and electrode separation) approaching, or in excess of, that for electrical breakdown at the particular value of E/p.

drift velocity by assuming, as one of several simplifications, that all electrons in the swarm have the same agitational speed. Subsequently it was shown (Townsend 1937) that, if proper account were taken of the distribution of speeds within the swarm, the drift velocities deduced from the simple formula had to be multiplied by a factor usually near unity, the magnitude of the factor depending on the form of energy distribution assumed.

Because of these difficulties of interpretation it is not surprising that the earlier method was superseded by the more direct time-of-flight techniques which date from those first used for electrons by Bradbury and Nielsen (1936). Although the limitations to the accuracy of the data obtainable with these techniques have not always been appreciated (Duncan 1957; Lowke 1962), it is in most circumstances possible in principle to produce data of high accuracy, whereas the method of magnetic deflection is susceptible to errors which can be as high as 50% or more (Section VI) but which remain unknown in the absence of information from other experiments.

With few exceptions (e.g. Hershey 1938; Hall 1955) the method has been abandoned for the measurement of electron and ion mobilities but an attempt was made by Huxley and Zaazou (1949) to exploit the technique as a means of investigating the distribution of electron energies in swarm experiments. At that time, as has already been pointed out, it was realized that the true drift velocity W could be obtained from the "drift velocity" W_M measured by a magnetic deflection technique only by multiplying by a dimensionless factor. Consequently, in their paper Huxley and Zaazou compared electron drift velocities measured by Bradbury and Nielsen (1936) with values of W_M measured by a new method based on Townsend's original method, and from this comparison drew some conclusions regarding the energy distributions of the electrons in the range of E/p over which the comparisons were made.

Subsequently, Huxley (1960) derived a rigorous relationship between W and W_M and showed that the coefficient C in the relation $W = CW_M$ depends directly on the variation of the momentum transfer cross section with electron energy, as well as indirectly through its effect on the distribution of electron energies. Application of the formula derived by Huxley (Sections II(c) and VI) shows, in fact, that the direct dependence may in some cases predominate in determining the value of the coefficient, so that comparisons such as those made by Huxley and Zaazou may not lead to significant information about the energy distribution function. This fact has also been pointed out by Phelps and his co-workers (Frost and Phelps 1962; Engelhardt, Phelps, and Risk 1964).

As a result of this later work it became clear that experimental results of high accuracy for the coefficient C were required if comparisons between theory and experiment were to be significant, and also that the range of electron energies for which data are available should be greatly extended. With the exception of the more recent work of Hall (1955) the only magnetic deflection experiments from which results are available are the early ones to which reference was made by Townsend (1948). The work described in the present paper was initiated to provide experimental results for W_M of accuracy comparable with those becoming available

for the more commonly measured transport coefficients, namely the drift velocity W and the coefficient of diffusion D. Huxley's method (Huxley and Zaazou 1949) has been used since it is more flexible in its application than Townsend's method and has the advantage that the theory upon which it is based does not rely on an approximation whose validity is questionable (Section II). The reliability of the method and the techniques used was established by following the procedure of Crompton and Jory (1962), that is by taking an extensive series of results for a wide range of those experimental parameters that can be conveniently varied, in this instance the pressure and the electric field and magnetic field strengths. Both the experiment itself, and the interpretation of the results from it in terms of W_M, are less straightforward than those for measuring W and W/D. Consequently, a critical examination of the method was undertaken when designing the experiment and an assessment made of the overall accuracy likely to be achieved.

For the reasons given in Section IV, nitrogen was chosen as the gas for this investigation. The coefficient C has been determined over the energy range corresponding to $0 \cdot 04 < E/p < 8 \cdot 0$ at 293°K and was found to vary by more than 50%. The reasons for this variation are discussed qualitatively in Section VI and a comparison is made between the new experimental results and the calculated values of Engelhardt, Phelps, and Risk (1964).

II. Theory

(a) Townsend's Method

The method used by Townsend (Townsend and Tizard 1913) for measuring drift velocities may be understood by reference to Figure 1. For a given value of gas pressure p and electric field strength E, the magnetic field B at right angles to E and parallel to the transverse cut in the receiving electrode was adjusted until equal currents were received by the two sections A and C. By assuming that the motion of the centre of the stream was along the resultant of the forces Ee and BeW, Townsend obtained the following relation between the drift velocity W, the electric and magnetic field strengths, and the angle θ which is determined by the apparatus:

$$W = (E/B)\tan \theta. \tag{1}$$

Subsequently, by taking account of the distribution of electron velocities, Townsend (1937) modified equation (1) by including a coefficient on the right-hand side whose upper and lower limits he found to be 4/3 and $0 \cdot 85$ corresponding respectively to a monoenergetic swarm and to a swarm with a Maxwellian energy distribution.

(b) Huxley's Method

A considerably more detailed analysis of the problem was made by Huxley (1937) who derived an expression for the ratio W_x/W_z of transverse to longitudinal drift velocity for a given magnetic field strength, in terms of the mean free paths and agitational speeds of the electrons taking into account the distribution of speeds. He showed that W_x/W_z could replace $\tan \theta$ in the formula given by Townsend but showed later (Huxley 1940) that, in fact, no simple relation exists between these two

quantities, although for small deflections W_x/W_z is approximately equal to $\tan \theta$. This led subsequently to the development of a new method (Huxley and Zaazou 1949) for measuring the ratio W_x/W_z directly without the use of the assumption inherent in Townsend's method. Furthermore, because the method no longer relies on a
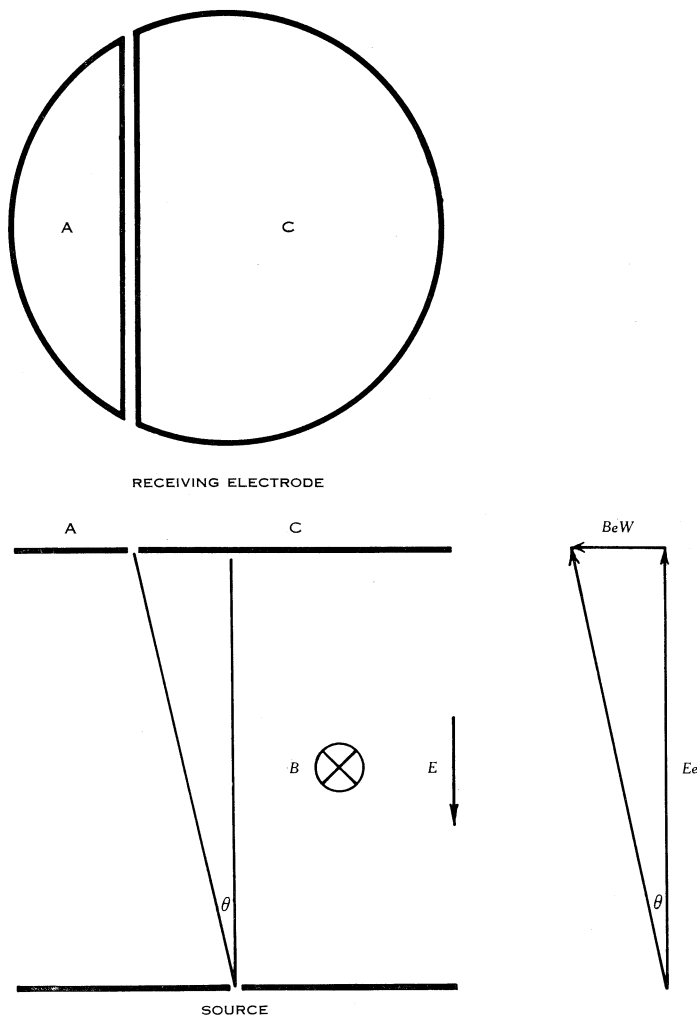

Fig. 1.—Schematic diagram of Townsend's drift-velocity apparatus.

geometrical property of the apparatus, namely the angle θ of Figure 1, it is much more flexible and allows measurements to be made at a given value of E/p for an infinite number of combinations of electric and magnetic field strengths. The method can therefore be subjected to much more stringent tests than the older method. It is this method which is used in the present work; the theory upon which it is based is outlined in Section II(d).

(c) The Coefficient C

(i) *Definition of* C *in Terms of Other Transport Coefficients.*—Huxley (1960) showed that the transport coefficients W and W_x/W_z were related through the equation

$$W = C(E/B)(W_x/W_z),$$

where C is a dimensionless factor which depends both on the variation of the momentum transfer cross section with electron energy and on the form of the energy distribution function.

By analogy with equation (1), if W_M is the "drift velocity" calculated from Townsend's original formula, then

$$W_M = (E/B)(W_x/W_z), \qquad\qquad (2)^*$$

from which it follows that

$$W = CW_M.$$

Since C is usually less than unity it is often more convenient to use the reciprocal of C, which will be denoted by ψ such that

$$\psi = C^{-1} = W_M/W$$

$$= (E/BW)(W_x/W_z). \qquad\qquad (3)$$

ψ has been termed the "magnetic deflection coefficient" by Frost and Phelps (1962) who used this quantity to compare the results of their theoretical calculations with experiment (Section VI).

Equation (3) shows that, in order to determine C or ψ experimentally, it is necessary to combine the results of experiments which measure the ratio W_x/W_z with values of W measured at the same value of E/p by a time-of-flight method.

(ii) *Calculation of* C *Theoretically.*—In the absence of a magnetic field it has been shown (e.g. Allis 1956; Huxley 1960) that the drift velocity in a constant and uniform electric field E parallel to the z axis can be calculated from the relation

$$W = W_z$$

$$= -\frac{4\pi Ee}{3Nm} \int_0^\infty \frac{c^2}{q_m} \frac{df}{dc} \, dc,$$

where N is the molecular number density, m is the electron mass, q_m is the momentum transfer cross section, and $f(c)$ is the function describing the distribution of electron speeds c. The application of a constant and uniform magnetic field B, at right angles to E and parallel to the y axis, results in a velocity no longer parallel to E but having a component at right angles to both E and B such that the resultant

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* It should be noted that equation (2) rather than equation (1) serves to define W_M when the deflection of the stream is sufficiently large for the approximation $W_x/W_z = \tan\theta$ to be invalidated.

velocity is given by (Huxley 1960)

$$W = W_z + iW_x$$

$$= -\frac{4\pi Ee}{3Nm} \int_0^\infty \frac{c^2}{q_m\{1-i(\omega/\nu_m)\}} \frac{df}{dc} \, dc,$$

where ν_m is the momentum transfer collision frequency and $\omega = Be/m$.

It follows that

$$W_z = -\frac{4\pi Ee}{3Nm} \int_0^\infty \frac{c^2}{q_m\{1+(\omega/\nu_m)^2\}} \frac{df}{dc} \, dc,$$

and

$$W_x = -\frac{4\pi Ee}{3Nm} \cdot \frac{Be}{Nm} \int_0^\infty \frac{c}{q_m^2\{1+(\omega/\nu_m)^2\}} \frac{df}{dc} \, dc.$$

In the limiting case of low magnetic field when $\omega/\nu_m \ll 1$ it follows that

$$\psi = \frac{E}{BW} \cdot \frac{W_x}{W_z}$$

$$\simeq \frac{EW_x}{BW^2}$$

$$= -\frac{3}{4\pi} \frac{\displaystyle\int_0^\infty \frac{c}{q_m^2} \frac{df}{dc} \, dc}{\left[\displaystyle\int_0^\infty \frac{c^2}{q_m} \frac{df}{dc} \, dc\right]^2}, \qquad (4)^*$$

so that the theoretical evaluation of ψ depends on a knowledge of the variation of q_m with c and of the distribution function $f(c)$.

From equation (4) it follows (Huxley 1960) that, for the special case of low energy electron swarms in the monatomic gases for which only elastic collisions need be considered, the value of ψ can be calculated for any given value of E/N provided the gas temperature is specified and the dependence of q_m on c is known, since the distribution function $f(c)$ is then completely specified. The calculation of ψ in other circumstances is a considerably more complex procedure (Frost and Phelps 1962).

(d) Theory of the Method of Measuring W_x/W_z

The method used in the present investigation (Huxley and Zaazou 1949) may be understood by reference to the schematic diagram of the apparatus shown in Figure 2. A uniform electric field parallel to the z axis is maintained between the electrodes by the application of appropriate potentials and a uniform magnetic field

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* Compare Huxley (1960) in which

$$C = 1/\psi = \tfrac{1}{3}[\overline{c^{-2} \, d(lc^2)/dc}]^2/[\overline{c^{-2} \, d(l^2c)/dc}],$$

where the bar denotes an average over the distribution function and $l = 1/Nq_m$ is the mean free path for momentum transfer.

is established parallel to the y axis. Electrons, having already reached a steady state in the region PS, enter the diffusion chamber through a small hole G in the cathode S and move towards the anode under the combined influence of the electric and magnetic fields. The receiving electrode ABCD is bisected by a slit parallel to the y axis.
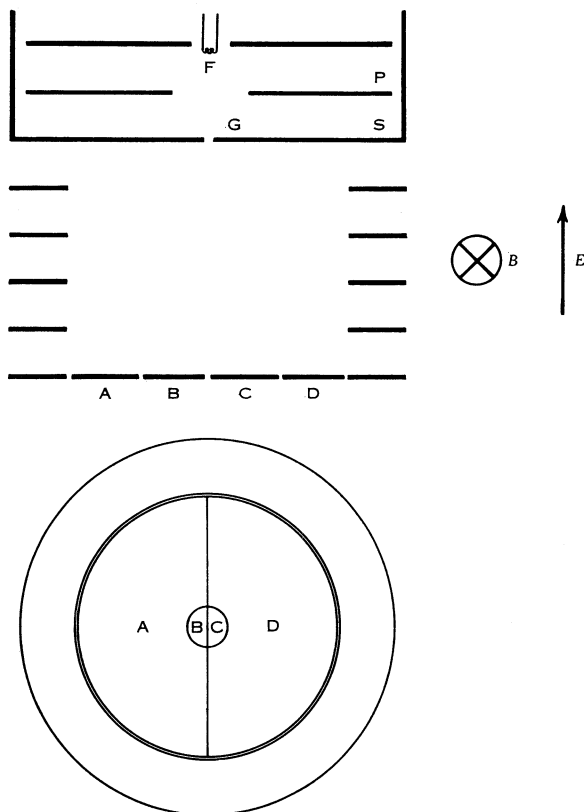

Fig. 2.—Schematic diagram of apparatus for measuring W/D and W_x/W_z.

In the absence of the magnetic field the currents received by each half of the receiving electrode are equal, but the application of the magnetic field deflects the diffusing stream so that the ratio R of the two currents will no longer be equal to unity. Following the method of Huxley* (Huxley and Zaazou 1949) the ratio R

* Equations (5) have been derived on the basis that the hole in the cathode acts as a pole source and that the electron concentration over the receiving electrode is zero (Huxley 1959). The same solution applies when the hole is regarded as a dipole source and the receiving electrode as a geometrical plane (Huxley and Zaazou 1949). As for the case with no magnetic field (Crompton and Jory 1962), h has been chosen to ensure that equations (5) are applicable also for the case of zero electron concentration at each electrode.

may be shown to be given by

$$R = \frac{1-A}{1+A},$$

where

$$A = \exp\left(-V\{1-(1-U)^{\frac{1}{2}}\}\right) \left(\frac{2UV}{\pi}\right)^{\frac{1}{2}} \sum_{n=0} \frac{1}{1.3.5.\ldots(2n+1)} (UV)^n F_n(V)$$

and

$$F_n(V) = 1 + \frac{(4n^2-1^2)}{8V} + \frac{(4n^2-1^2)(4n^2-3^2)}{(8V)^2 2!} + \ldots,$$

$$V = \tfrac{1}{2}(W/D)h\{1+(W_x/W_z)^2\}^{\frac{1}{2}}\{1+4(W_x/W_z)^2 r\},$$

$$U = \frac{(W_x/W_z)^2}{\{1+(W_x/W_z)^2\}} = \frac{W_x^2}{W_x^2+W_z^2},$$

(5)

where h is the length of the diffusion chamber.

The dimensionless quantity r is a function of the electron energy distribution and of the variation of q_m with c. Calculation of the values of R at the widest possible extremes of the value of r has shown that the maximum discrepancy in R will be $0\cdot5\%$. However, small variations in the value of r will have little or no effect on the values of W_x/W_z calculated from the measured values of R.

The equations (5) show that R is a function both of the ratio W/D and of the ratio W_x/W_z. The ratio W/D is either known from independent measurement or, as in this instance, can be determined using the same apparatus. For this purpose the receiving electrode is also divided by a circular cut of known radius so that by connecting appropriate segments of the electrode together (that is B to C and A to D in Fig. 2) the ratio of the current received by the central disk to the total current can be measured in the absence of the magnetic field. The ratio W/D can be calculated from this current ratio in the usual way (Crompton and Jory 1962). From the results of the two experiments W_x/W_z can then be calculated.

III. APPARATUS

A diagram of the apparatus used in the present work is shown in Figure 3. Electrons emitted thermionically from the filament F pass through a hole G in the centre of the cathode S and drift towards the receiving electrode H, which consists of a central disk and surrounding annulus concentric with the axis of the diffusion chamber, both of which are bisected by a transverse slit. The construction of the apparatus is similar to that used by Crompton and Elford (1963), the significant dimensions being as follows:

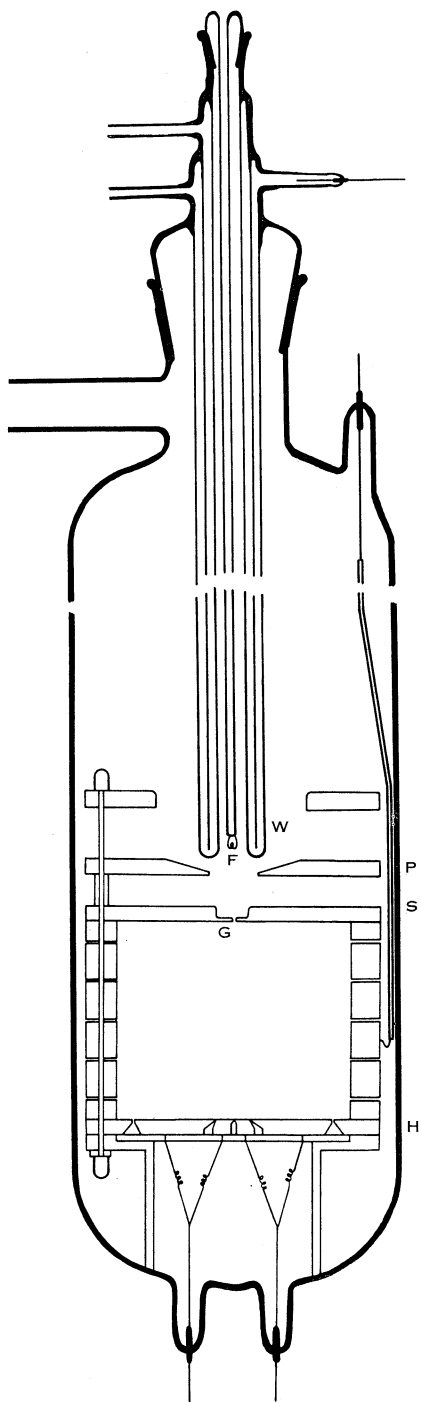|	
length	$83\cdot34\pm0\cdot04$ mm
thickness of guard electrodes	$16\cdot16\pm0\cdot005$ mm
thickness of glass spacers	$0\cdot508\pm0\cdot005$ mm
effective radius of central disk	$6\cdot00\pm0\cdot01$ mm
width of gap between disk and annulus	$0\cdot29\pm0\cdot1$ mm
width of transverse cut	$0\cdot25\pm0\cdot05$ mm

Fig. 3.—Diagram of diffusion apparatus.

diameter of source hole $1 \cdot 0$ mm
inner diameter of guard electrodes   100 mm.

The filament, wound from $0 \cdot 002$ in. diam. platinum wire, is surrounded by a glass cooling jacket W through which water can be circulated to remove the bulk of the heat generated by the filament. Surfaces adjacent to the filament are coated with platinum to serve as an additional electrode for the control of the electron current incident on S. The electric field in the region between the electrodes P and S is the same as that between S and H to ensure that the energy distribution of the electrons entering the diffusion chamber through G is substantially the same as within the chamber itself.

Each segment of the receiving electrode is mounted on a sheet of plate glass which is in turn mounted on the outer ring of H. This method of construction maintains adequate electrical resistance between each segment and also the degree of mechanical stability necessary to eliminate distortion of the whole assembly. Improved insulation between adjacent segments, together with reduced interelectrode capacitance, is obtained by undercutting the electrodes as shown in the diagram.

All guard electrodes, the cathode surface, and the surface of the assembled receiving electrode were lapped and polished to obtain a high surface finish and to ensure that the electrode faces were accurately parallel. To reduce contact potential differences to a minimum all surfaces presented to the diffusing electron beam were gold plated. Alignment of the whole apparatus was achieved by means of a Taylor–Hobson alignment telescope, whereby the coplanarity of all electrode surfaces and the accurate positioning of the centres of the source and receiving electrodes on the axis of the apparatus were assured.

The form of construction of the glass envelope, which contained three demountable seals, precluded thermal outgassing of the apparatus. Nevertheless, the use of metal and glass only in the construction of the apparatus, coupled with attention to surface finish on all metal components and long preliminary periods under vacuum, resulted in an outgassing rate entirely acceptable in these experiments. Without use of liquid nitrogen traps the pressure rise in the whole system was found to be less than 3×10^{-4} torr/24 hr (the maximum time for any experimental run); this figure was reduced to less than 2×10^{-4} torr/24 hr with the use of liquid nitrogen traps. A 5 l/s getter-ion pump was used to produce an initial vacuum of less than 10^{-6} torr before the commencement of any experimental run.

Matheson "research-grade" gas was obtained from a cylinder through a Matheson "ultra-pure transfer regulator" and a conventional metal UHV tap arranged so that the gas admittance system could be evacuated up to the stopcock of the cylinder. The gas was passed through a liquid nitrogen trap immediately adjacent to the experimental tube; no further purification was considered necessary.

Gas pressures were measured using precision capsule gauges as described by Crompton and Elford (1957) which were modified to the extent of replacing the "O" ring seal by a seal employing Apiezon W40 wax.

The ratios of currents received by the segments of the receiving electrode were measured using the equipment described by Crompton and Jory (1962), which enables

ratios of currents as small as 5×10^{-13} A to be measured to within $0 \cdot 1\%$ for nearly equal currents while maintaining all sections of the receiving electrode to within $0 \cdot 2$ mV of earth potential.

Magnetic fields were generated by a pair of Helmholtz coils of 26 cm diam., designed to achieve maximum uniformity of field as described by Barker (1949). Subsequent measurements with a differential gaussmeter showed that the variation of field strength within the volume occupied by the diffusing electron stream (Section IV) was less than $0 \cdot 1\%$.

The constant of proportionality relating the field strength to the current flowing through the coils, which was used throughout in determining B, was calculated using the formula quoted by Barker and checked with the gaussmeter. Agreement between the two methods was better than 1%. The current flowing through the coils was supplied by a stabilized d.c. power supply and measured with a standard $0 \cdot 01$ ohm resistor and differential voltmeter.

IV. Experimental Method

Equations (3) and (5) show that several independent variables must be either known or measured in order to calculate the coefficient ψ. Errors in the determination of each of these quantities have therefore a cumulative effect in the determination of ψ; it was for this reason amongst others that nitrogen was chosen as the gas to establish the reliability of the experimental methods, since the determination of two of these variables, the drift velocity W (Lowke 1963) and the ratio W/D (Crompton and Elford 1963), has been the subject of considerable investigation in this laboratory. The availability of the data for nitrogen, together with the fact that transport coefficients in polyatomic gases are less sensitive to minute traces of impurity than those in the monatomic gases, weighed against the first measurements being made in a monatomic gas for which the theoretical evaluation of ψ through equation (4) is a comparatively straightforward procedure. Nitrogen was chosen in preference to hydrogen because the theoretical predictions of Frost and Phelps (1962) show a large variation of ψ with E/p in a convenient range of measurement. Although no attempt was made to improve on Lowke's drift velocity data, measurements of the ratio W/D were made with the present apparatus for two reasons. First, a comparison of these results with those of Crompton and Elford serves as a check on the accuracy of the diffusion apparatus, since the earlier results were taken with an apparatus with which it is possible to achieve higher accuracy because of the simpler configuration of the receiving electrode. Secondly, when calculating W_x/W_z from equation (5), it seems more appropriate to use the value of W/D determined for the actual combination of gas pressure and electric field strength used when measuring the deflection of the stream in the magnetic field, rather than an average value taken from published data.

The experimental procedure was therefore as follows. For a given gas pressure, measurements were made in the absence of the magnetic field with the leads from the segments of the collecting electrode connected together in such a way as to form a central disk and surrounding annulus. For a range of values of electric field strength

E, restricted as described below, measurements of the ratio of currents received by the disk and annulus were made and from these measurements values of W/D determined as described, for example, by Crompton and Jory (1962). The leads were then rearranged in such a way that the collecting electrode took the form of two semicircular segments and the ratio of the currents received by these segments was measured at each value of E for a range of values of the magnetic field strength B. Errors introduced by inaccuracies of alignment in the apparatus, in this experiment of greater significance than in experiments to determine W/D, were minimized by averaging the results obtained for the forward and reverse directions of the magnetic field. The horizontal component of the Earth's magnetic field was measured and added vectorially to the main field in each instance. The data were then analysed using a procedure similar to that described by Huxley and Zaazou (1949) to yield values of W_x/W_z.

In order to examine the consistency of the method the experimental parameters were varied over as wide a range as possible while still remaining within limitations similar to those discussed by Crompton and Jory (1962). These parameters were:

(i) *Electric Field Strength* E.—Field strengths in the range 3 V/cm to 40 V/cm were used. The lower limit was determined by the onset of significant errors from contact potential differences. Under some experimental conditions, the lower limit was raised by the need to restrict the divergence of the electron stream to prevent a significant proportion of it from entering the region of insufficiently uniform field adjacent to the guard electrodes (see below).

(ii) *Gas Pressure* p.—Pressures of 5, 10, 20, 50, 100, and 200 torr were used and, except for the lowest pressures, could be set to within 1%.

(iii) *Magnetic Field Strength* B.—The values of B used were nominally 20, 40, 60, 80, 100, and 120 gauss. Actual values differed somewhat from the nominal values because of lack of resolution in the power supply but these were determined in each case by measuring the current through the coils.

A given combination of p, E, and B was used only if the current ratios R could be measured with sufficient accuracy to enable the values of W/D and W_x/W_z to be determined to within $0 \cdot 5\%$ or better and provided that no more than $0 \cdot 1\%$ of the electron stream arriving at the receiving electrode fell outside the cylinder of radius 3 cm within which the electric field was adequately uniform.

(iv) *Temperature*.—The use of the water cooling jacket ensured that the gas temperature remained within 1 degC of the ambient laboratory temperature of 293°K (Crompton and Elford 1963). As there was no provision in this apparatus for measuring the gas temperature directly, no correction for variation in temperature was made, but these corrections would amount to less than $0 \cdot 5\%$.

The Matheson research-grade nitrogen used in these experiments was guaranteed to have a total impurity level of less than 25 p.p.m., of which 15 p.p.m. were mona-tomic gases whose influence on the measured transport coefficients would be negligible. The outgassing rates quoted in Section III would raise the total level to a maximum

value of 50 p.p.m. for a sample of gas at a pressure of 5 torr kept for 24 hr, while the high pressure samples would be insignificantly contaminated over this period. Confirmation of the insignificance of these levels is afforded by the absence of a variation with time of the results taken at any pressure.

TABLE 1

VALUES OF k_1 IN NITROGEN AT 293°K

p (torr) E/p (V cm^{-1} torr^{-1})	200	100	50	20	10	5	Crompton and Elford
0·04	1·90						1·87
0·05	2·25						2·23
0·06	2·62	2·60					2·60
0·07	3·01	2·97					2·98
0·08	3·39	3·34					3·36
0·09	3·74	3·71					3·74
0·10	4·12	4·06					4·10
0·12	4·77	4·74	4·79				
0·15	5·78	5·69	5·75	5·73			5·75
0·18	6·70	6·60	6·65	6·64			
0·20	7·30	7·19	7·24	7·20			7·25
0·25		8·65	8·71	8·66			
0·3		10·13	10·18	10·08			10·12
0·4		13·00	13·05	12·97			12·96
0·5			15·66	15·62			15·56
0·6			17·98	17·88			17·80
0·7			19·92	19·91			19·70
0·8			21·6	21·5	21·0		21·4
0·9				22·9	22·5		22·9
1·0				24·2	23·7		24·1
1·2				26·3	25·9		
1·5				28·7	28·4		28·7
1·8				30·7	30·4		
2·0				31·8	31·5		31·7
2·5					33·9	33·4	
3·0					35·9	35·4	36·0
4·0					39·0	38·6	39·0
5·0						41·1	41·3
6·0						43·0	
7·0						44·5	
8·0						45·7	

V. RESULTS

(a) Results for the Ratio W/D

When results taken at the same temperature but at different pressures are to be compared it is convenient to present them in terms of the energy factor k_1 (cf. Huxley and Crompton 1962). The following relations can be used to calculate the

TABLE 2

EXPERIMENTAL VALUES OF $W_M \times 10^{-5}$ (CM/S) IN NITROGEN AT 293°K

Notes on structure: the three data groups each carry two pressures according to the E/p range — Group 1 = $p = 200$ torr (upper rows) / $p = 20$ torr (lower rows); Group 2 = $p = 100$ torr / $p = 10$ torr; Group 3 = $p = 50$ torr / $p = 5$ torr. Group 3 shows only $B = 20, 40, 100$. A note "(B = 120)" appears in the $p = 20$ torr block.

E/p (V cm⁻¹ torr⁻¹)	20	40	60	80	100	20	40	60	80	100	20	40	100	Least Value	Greatest Value	Best Estimate
0.04	3·94	3·94												3·94	3·94	3·94
0.05	4·11	4·15	4·18											4·11	4·18	4·15
0.06	4·23	4·27	4·28			4·28	4·24							4·23	4·28	4·27
0.07	4·28	4·36	4·38	4·41		4·33	4·30	4·28						4·28	4·41	4·35
0.08	4·34	4·41	4·41	4·46		4·40	4·35	4·36						4·34	4·41	4·38
0.09	4·39	4·46	4·46	4·51	4·53	4·46	4·40	4·39						4·39	4·51	4·45
0.10	4·46	4·50	4·49	4·55	4·58	4·49	4·46	4·42	4·39		4·67	4·67		4·42	4·65	4·4(8)
0.12	4·51	4·58	4·57	4·61	4·64	4·56	4·56	4·52	4·50		4·82	4·81		4·50	4·64	4·5(7)
0.15		4·76	4·76	4·80	4·85	4·75	4·73	4·70	4·68	4·67	4·98	4·96		4·68	4·85	4·7(3)
0.18		4·93	4·95	4·97	4·91*	4·92	4·92	4·88	4·83	4·88	5·11	5·07		4·83	4·97	4·9(3)
0.20		5·03	5·05	5·07	5·10	5·00	5·00	4·97	4·96	4·96	5·36	5·33	5·32	4·96	5·11	5·0(4)
0.25						5·31	5·21	5·24	5·22	5·23	5·52	5·54	5·56	5·21	5·36	5·3(0)
0.30		5·84	5·81	5·79		5·48	5·45	5·48	5·44	5·43	5·92	5·91	5·90	5·43	5·56	5·5(0)
0.40		6·20	6·17	6·18			5·83	5·87	5·82	5·82		6·26	6·30	5·79	5·92	5·8(7)
0.50		6·57	6·57	6·60								6·80	6·72	6·17	6·30	6·2(0)
0.60	6·57	6·58	6·57		6·60							7·06	7·10	6·57	6·72	6·5(7)
0.70	7·00	7·00	7·01	7·01	7·02	7·30	7·30	7·29				7·52	7·50	7·00	7·10	7·0(5)
0.80	7·45	7·41	7·46	7·46	7·45	7·84	7·83	7·76						7·29	7·52	7·4(0)
0.90	7·89	7·86	7·94	7·90	7·91	8·27	8·27	8·23						7·76	7·94	7·8(5)
1.00	8·41	8·47	8·41	8·49	8·40	9·25	9·25	9·25						8·23	8·49	8·4(0)
1.20	9·37	9·36	9·40	9·39	9·37	10·79	10·73	10·75	10·76					9·25	9·40	9·3(5)
1.50	11·01	10·88	10·84	10·88	10·92	12·19	12·23	12·26	12·23					10·73	11·01	10·8
1.80	12·38	12·35	12·45	12·41	12·42	13·19	13·26	13·21	13·26					12·19	12·42	12·3
2.00	13·38	13·42	13·41	13·38	13·39									13·19	13·42	13·3
2.50						15·5	15·6	15·6	15·7		15·6	15·5		15·5	15·7	15·6
3.00						18·0	18·0	17·7	18·0		17·8	17·9		17·7	18·0	17·9
4.00						22·5	22·4	22·6	22·6		22·5	22·5		22·4	22·6	22·5
5.00											27·4	27·0		27·0	27·4	27·0
6.00											31·5	31·6		31·5	31·6	31·6
7.00											35·9	36·1		35·9	36·1	36·0
8.00											40·0	40·2		40·0	40·2	40·1

* Values shown in italics are not considered reliable.

quantity W/D and the "characteristic energy" D/μ (as used by Phelps and co-workers) from values of k_1 measured at $T°$K:

$$W/D = (e/kT)\,(E/k_1),$$
$$D/\mu = k_1(e/kT)^{-1},$$

(6)

where μ is defined as the ratio of the drift velocity W to the electric field strength E.

The results for nitrogen at 293°K obtained at pressures used in these experiments are given in Table 1 for $0\cdot04 < E/p < 8\cdot0$ V cm^{-1} torr^{-1}. Except for one measurement made at the lowest value of E/p the results are internally consistent and agree with those of Crompton and Elford (1963) to within 1%. Since the addition of the transverse cut in the receiving electrode adds considerably to the difficulty of maintaining adequate mechanical tolerances in this critical region the agreement shown in the table is satisfactory.

(b) Results for the Ratio W_x/W_z

Since the ratio W_x/W_z is a function of both E/p and B/p, a comparison of the experimental results taken at the same value of E/p but at different pressures or with different magnetic field strengths is not easily made. A quantity which is invariant* with p and B is the quantity $W_M = (E/B)\,(W_x/W_z)$, so that values of W_M rather than W_x/W_z are presented in Table 2. As has been already pointed out the values of magnetic field strength are nominal values only although it was necessary, of course, to use the actual values to calculate W_M.

The last three columns of the table show, respectively, the largest and smallest values of W_M obtained at any given value of E/p, and the best estimate obtained by biasing the result towards those for which the experimental conditions allow maximum accuracy. The maximum scatter for the data at any given value of E/p is seen to be of the order of $\pm2\%$, which is considered satisfactory in view of the range of values of p and B used and the number of independent variables which must be measured for their determination. It should be remembered that these values may also be subject to a systematic error of not more than 1% arising from a possible error in the constant of proportionality between B and the current flowing through the Helmholtz coils (Section III).

(c) Results for the Coefficient $\psi = 1/C$

The results for W_x/W_z given in the final column of Table 2 have been used in conjunction with Lowke's (1963) time-of-flight drift velocity data to calculate values of the coefficient ψ. The results are presented as a function of E/p in Table 3 and in Figure 4. Apart from the possible systematic error discussed above, the values of ψ are subject to the combined errors of the measurements of W_M and W, amounting probably to no more than 2–3%.

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* This applies only to the limiting case of low magnetic field strength to which this work is restricted.

VI. Discussion

The results given in Table 2 show that, despite the difficulties involved, it is possible to determine the ratio W_x/W_z of the transverse to longitudinal drift velocities in crossed electric and magnetic fields with an accuracy comparable to that now attainable for data for W/D and W (Crompton and Elford 1963; Lowke 1963).

TABLE 3

COMPARISON OF CALCULATED AND EXPERIMENTAL VALUES OF ψ IN NITROGEN AT 293°K

E/p (V cm^{-1} torr^{-1})	$W_M \times 10^{-5}$ (cm/s) [From Table 2]	$W \times 10^{-5}$ (cm/s) [Lowke]	ψ		
			[From W_M (Table 2) and Lowke]	[From W_M (Townsend and Bailey) and Lowke]	[Engelhardt, Phelps, and Risk]
0·04	3·94	2·55	1·55		1·61
0·05	4·15	2·71	1·53		1·59
0·06	4·27	2·81	1·52		
0·07	4·35	2·90	1·50		
0·08	4·38	2·96	1·48		1·49
0·09	4·45	3·03	1·47		
0·10	4·4(8)	3·09	1·45		1·45
0·12	4·5(7)	3·22	1·42		
0·15	4·7(3)	3·43	1·38		1·38
0·18	4·9(3)	3·63	1·36		
0·20	5·0(4)	3·76	1·34		1·34
0·25	5·3(0)	4·02	1·32		1·30
0·30	5·5(0)	4·28	1·28	1·24	1·27
0·40	5·8(7)	4·76	1·23	1·23	1·23
0·50	6·2(0)	5·19	1·20	1·19	1·19
0·60	6·5(7)	5·68	1·16	1·18	
0·70	7·0(5)	6·18	1·14	1·17	
0·80	7·4(0)	6·66	1·11	1·14	1·13
0·90	7·8(5)	7·19	1·09	1·13	
1·00	8·4(0)	7·72	1·09	1·11	1·10
1·20	9·3(5)	8·73	1·07		
1·50	10·8	10·2	1·06	1·07	1·06
1·80	12·3	11·7	1·05		
2·00	13·3	12·7	1·05	1·03	1·05
2·50	15·6	14·9	1·05	1·05	1·04
3·00	17·9	17·1	1·05	1·04	1·05
4·00	22·5	21·1	1·07		1·07
5·00	27·0	25·0	1·08	1·08	1·08
6·00	31·6	28·8	1·10		
7·00	36·0	32·3	1·11		
8·00	40·1	35·7	1·12		1·14

Results of this accuracy allow the coefficient ψ to be calculated with only slightly inferior accuracy so that this coefficient may now be used, together with those more frequently employed, to interpret the macroscopic behaviour of electron swarms in terms of microscopic collision processes. The new data also extend considerably the range of E/p for which data of this kind are available.

It has already been pointed out that the coefficient ψ is sensitive both to the variation of the momentum transfer cross section q_m with electron energy and to the form of the energy distribution function. For electron swarms in the monatomic gases in the energy regime in which only elastic collisions need be considered, the form of the function is itself predominantly determined by the dependence of q_m on electron speed c provided the energy factor k_1 is very much greater than unity (Huxley 1960). The value of ψ is then uniquely determined by this dependence (Section II(c)). Such is not the case, however, for the polyatomic gases. In nitrogen at 77°K, for example,
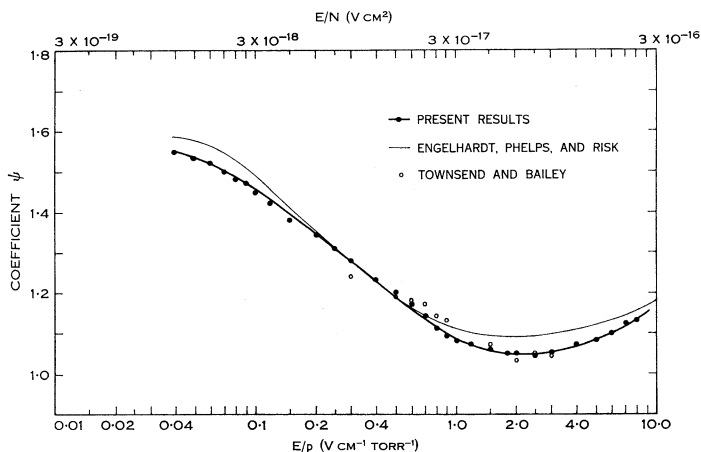
Fig. 4.—The variation of the coefficient ψ with E/p in nitrogen at 293°K as determined from the present experiments, from the experiments of Townsend and Bailey, and from the theoretical calculations of Engelhardt, Phelps, and Risk. In each case Lowke's values of drift velocity have been used.

Engelhardt, Phelps, and Risk (1964) have calculated that, for k_1 less than about 10, the energy distribution lies between the Maxwellian and Druyvesteyn forms. Were it not for the influence of inelastic collisions, which in this regime are those responsible for rotational excitation, the distribution would be expected to become narrower than either of these distributions as $k_1 \to 10$ since it is known that q_m is approximately proportional to c (Huxley and Crompton 1962; Engelhardt, Phelps, and Risk 1964) for a large fraction of the electrons in a swarm with this mean energy. It is profitable, therefore, to use equation (4) to calculate values of ψ for various dependences of q_m on c and for various typical (although ideal) energy distribution functions (cf. Frost and Phelps 1962; Huxley and Crompton 1962) and to compare the calculated values with those measured experimentally. Values of ψ calculated in this way are shown in Table 4. In calculating the values of ψ the dependence of q_m on c has been represented simply by

$$q_m = \text{const.}\ c^r$$

and the distribution function for the electron speeds by

$$f(c) = \text{const.}\ \exp\{(-c/a)^n\}, \tag{7}$$

particular cases of which are the Maxwellian distribution ($n = 2$) and the Druyvesteyn distribution ($n = 4$). When only elastic collisions need be considered it can be shown (e.g. Allis 1956; Huxley and Crompton 1962) that the distribution function appropriate to $q_m = \text{const. } c^r$ is that given by equation (7) with $n = 4+2r$.

The lowest value of E/p for which experimental results are given in Table 3 corresponds to a mean electron energy of approximately $0\cdot 08$ eV or about twice thermal energy at 293°K. In this region q_m is approximately proportional to c for the majority of the electrons; furthermore, the distribution is approaching the Maxwellian form both because of the proximity to the thermal region and because of the influence of inelastic collisions. The value of ψ measured at the lowest value of E/p is approximately $1\cdot 6$; the departure of this value from the value $3\cdot 0$ corresponding to constant q_m and a Maxwellian distribution is probably predominantly a measure of the departure of the distribution from the Maxwellian form (cf. Engelhardt, Phelps, and Risk 1964) for measurements made at this temperature (293°K).

TABLE 4

COMPUTED VALUES OF ψ

n \ r	-1	0	1
2 (Maxwell)	$1\cdot 0$	$1\cdot 18$	$3\cdot 0$
4 (Druyvesteyn)	$1\cdot 0$	$1\cdot 06$	$1\cdot 38$
6	$1\cdot 0$	$1\cdot 03$	$1\cdot 16$

A qualitative explanation of the steady decrease in ψ with increasing E/p can also be given by referring to Table 4. As E/p increases the energy distribution departs further from the Maxwellian form and, furthermore, a significant number of electrons in the swarm are now in the energy range for which q_m is more nearly independent of energy (Engelhardt, Phelps, and Risk 1964). Both factors tend to lower the value of ψ. The value at the minimum suggests, in fact, that the distribution is here somewhat narrower than the Druyvesteyn form. This would be expected if the inelastic collisions had no effect on the energy distribution. For still higher values of E/p, ψ again rises presumably owing to the contribution of those electrons with energy above 1 eV for which q_m varies rather strongly with c.

Although the qualitative discussion above is useful in interpreting the variation of ψ with E/p, a complete explanation cannot be given for any polyatomic gas without a detailed analysis of the problem taking proper account of the modification to the energy distribution function produced by the inelastic collisions. Such an analysis for nitrogen has recently been published by Engelhardt, Phelps, and Risk (1964)

who extended the earlier work of Frost and Phelps (1962). These authors have attempted, with considerable success, to find sets of energy-dependent elastic and inelastic cross sections that are consistent with published data for electron drift velocities and diffusion coefficients. As a further check on the success of their analysis they have compared values of ψ which they have calculated theoretically with experimental values. The experimental values were obtained by combining the somewhat limited data for W_M published by Townsend and Bailey (1921) with Lowke's (1963) values of W determined by the Bradbury and Nielsen time-of-flight method.

The values of ψ calculated by Engelhardt, Phelps, and Risk are plotted in Figure 4 for comparison with the new experimental results. The agreement is generally good, although the discrepancies between the two curves at low E/p and in the vicinity of $E/p = 2\cdot0$ are outside experimental error. It should be noted, however, that the experimental error assigned does not allow for the possibility of error in the theoretical interpretation of the magnetic deflection measurements upon which the calculation of ψ is based, although the good agreement between the results given in Section V for a large range of values of p and B suggests that this is not so. In this context it should be noted that errors in the time-of-flight values of W used in calculating $\psi = W_M/W$ would in this instance affect both curves equally since Lowke's data were used to form this ratio in both cases. The discrepancy which exists requires further examination.

Finally, it is interesting to note the very considerable errors which can be made in some circumstances in taking the values of W_M from magnetic deflection measurements as the true drift velocity W. Although little error is made if the collision frequency is approximately constant, Table 4 shows that errors of 50% or more can be incurred where q_m varies markedly with c and where the distribution function approaches the Maxwellian form as, for example, where the mean energy of the swarm approaches $\frac{3}{2}\,kT$. While earlier attempts to deduce W from W_M more accurately by allowing for the energy distribution in the swarm had partial success, leading to correction factors of the order of 10–20% (cf. Huxley and Zaazou 1949, and references therein), the much more pronounced influence of a strong variation of q_m with c was not pointed out until quite recently (Huxley 1960). The results from the present experiments in nitrogen demonstrate clearly that drift velocities deduced from magnetic deflection measurements should not be considered if corresponding data from time-of-flight experiments are available.

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