AMBIPOLAR DIFFUSION IN METEOR TRAILS

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Summary

Ambipolar diffusion and the formulation of the equations of continuity for a diffusing cloud of electrons and ions are briefly discussed. A numerical method for the solution of the equations is outlined and the solutions are presented in graphical form. The results indicate that, for concentrations higher than a critical value, electrons and ions diffuse at the same rate and that the major part of the radio echo from a meteor trail will occur under conditions of ambipolar diffusion. However, a sheath of electrons is to be expected surrounding the diffusing cloud. The ratio mobility: diffusion coefficient is shown to be an important parameter in controlling the transition from ambipolar to free diffusion.

I. Introduction

Ambipolar diffusion is discussed frequently in the literature but mostly in general terms. The effect is described as occurring in an ionized gas consisting of, say, electrons and singly charged ions. The large difference in the diffusion coefficients for the two particles will lead to their separation. The separation of the charged particles will produce electrostatic forces tending to oppose the separation. As a result the electron diffusion rate is decreased while that for the ions is increased and both particles diffuse at the same rate given by the coefficient of ambipolar diffusion D_{A}. Huxley (1952) has shown that D_{A} can be given in terms of the simple electron and ion diffusion coefficients and the particle mobilities, and Kaiser (1953) shows that in meteor trails $D_{\mathrm{A}} = 2D_{\mathrm{i}}$. In each of these papers it is assumed that the electron and ion concentrations n_{e} and n_{i} remain substantially equal throughout the process. In the present paper it is shown to what extent this remains true.

II. The Diffusion Equation

The existence of electrostatic forces in the diffusing cloud can lead to difficulties in the formulation of the appropriate equations. This is due to the fact that the electrostatic forces follow an inverse square law and so vary with distance between the particles only very slowly compared with the forces of interaction between the particles themselves. This means that two particles involved in a collision may each be under the influence of many other particles owing to the distant action of the electrostatic forces. Thus encounters between particles tend to become multiple and the simple diffusion theory, based on binary encounters, may not apply.

On the other hand, in considering the force exerted on a given particle by other particles, it is apparent that the force due to particles in the immediate neighbourhood varies rapidly in magnitude and direction as the particles move

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about, but the resultant force exerted by particles at distances large compared with the mean distance between neighbouring particles varies slowly and may be replaced by the force due to a continuous distribution of charge. This can be regarded as a body force or force causing a drift of all particles rather than a force of encounter between particles.

The particle currents are then composed of a diffusion current under the action of a concentration gradient and a drift current due to a local electrostatic field.

The currents per unit area are given by

$$\bar{J}_{\mathrm{e}} = -D_{\mathrm{e}}\nabla n_{\mathrm{e}}+n_{\mathrm{e}}\,\overline{W}_{\mathrm{e}}, \tag{1}$$

$$\bar{J}_{\mathrm{i}} = -D_{\mathrm{i}}\nabla n_{\mathrm{i}}-n_{\mathrm{i}}\,\overline{W}_{\mathrm{i}}, \tag{2}$$

where n = number density,
D = diffusion coefficient,
$\overline{W}$ = drift velocity in electric field E, and subscripts e and i refer to electrons and ions.

It has been shown by Kaiser (1953) that the processes of recombination and attachment are relatively unimportant in meteor trails and these effects are neglected here. Hence,

$$\nabla.\bar{J} = -\partial n/\partial t. \tag{3}$$

Therefore (1) and (2) in conjunction with (3) give:

$$\partial n_{\mathrm{e}}/\partial t = D_{\mathrm{e}}\,\nabla^2 n_{\mathrm{e}}-\nabla.(n_{\mathrm{e}}\overline{W}_{\mathrm{e}}), \tag{4}$$

$$\partial n_{\mathrm{i}}/\partial t = D_{\mathrm{i}}\,\nabla^2 n_{\mathrm{i}}+\nabla.(n_{\mathrm{i}}\,\overline{W}_{\mathrm{i}}). \tag{5}$$

In addition the electrostatic field is given by

$$\nabla.\bar{E} = (n_{\mathrm{e}}-n_{\mathrm{i}})\,e/\epsilon_0, \tag{6}$$

where e is the electronic charge and ϵ_0 is the permittivity of free space.

III. Method of Solution

In the remainder of this paper a numerical solution of equations (4) and (5) will be described which represents the diffusion of a cylindrical column of ions and electrons produced by a passing meteor. Number densities are quoted as volume densities. Since $\overline{W} = M\bar{E}$, where M = mobility, (4) can be written,

$$\partial n_{\mathrm{e}}/\partial t = D_{\mathrm{e}}\,\nabla^2 n_{\mathrm{e}}-M_{\mathrm{e}}\,\nabla.(n_{\mathrm{e}}\,\bar{E}). \tag{7}$$

It is convenient to rewrite the equations in dimensionless form using the parameters $\tau = tD_{\mathrm{e}}/a^2$; $\zeta = x/a$, where a is a typical length for the system; $\lambda = (M_{\mathrm{e}}^2 e/D_{\mathrm{e}}^2\epsilon_0).E$; $\alpha = M_{\mathrm{e}}e/aD_{\mathrm{e}}\epsilon_0$.

Thus (7) may be written

$$\frac{\partial n_{\mathrm{e}}}{\partial \tau} = \nabla^2 n_{\mathrm{e}}-\frac{1}{\alpha}\,\nabla.(n_{\mathrm{e}}\,\lambda), \tag{8}$$

in which the space derivatives are taken with respect to the variable ζ.

Likewise (6) becomes

$$\frac{D_e^2\epsilon_0}{M_e^2 e}\cdot\frac{1}{a}\,\nabla.\lambda = (n_e-n_i)\,\frac{e}{\epsilon_0},$$

whence

$$\begin{aligned}\nabla.\lambda &= (n_e-n_i)(e^2M_e^2a/\epsilon_0^2D_e^2)\\ &= \alpha^2a^3(n_e-n_i),\end{aligned}$$

and

$$\lambda = -\alpha^2a^3\,\frac{1}{\zeta}\int^{\zeta}\mathrm{d}\zeta'\zeta'(n_e-n_i),$$

assuming cylindrical symmetry.

Hence

$$\frac{\partial n_e}{\partial\tau} = \frac{1}{\zeta}\frac{\partial}{\partial\zeta}\left(\zeta\,\frac{\partial n_e}{\partial\zeta}\right)-\alpha a^3n_e(n_e-n_i)\,\frac{\partial n_e}{\partial\zeta}\cdot\frac{\lambda}{\alpha},$$

that is,

$$\frac{\partial n_e}{\partial\tau} = \frac{\partial^2n_e}{\partial\zeta^2}+\frac{1}{\zeta}\cdot\frac{\partial n_e}{\partial\zeta}-\,\alpha a^3n_e(n_e-n_i)+\frac{\partial n_e}{\partial\zeta}\cdot\frac{\alpha a^3}{\zeta}\int^{\zeta}\mathrm{d}\zeta'\zeta'(n_e-n_i). \tag{9}$$

In the same way the equation for ions becomes

$$\frac{\partial n_i}{\partial\tau} = \beta\,\frac{\partial^2n_i}{\partial\zeta^2}+\frac{\beta}{\zeta}\frac{\partial n_i}{\partial\zeta}+\gamma\alpha a^3n_i(n_e-n_i)-\frac{\partial n_i}{\partial\zeta}\cdot\gamma\frac{\alpha a^3}{\zeta}\int^{\zeta}\mathrm{d}\zeta'\zeta'(n_e-n_i), \tag{10}$$

where $\beta = D_i/D_e$, $\gamma = M_i/M_e$.

The ratio γ is shown by Huxley (1952) to be a small quantity of magnitude 10^{-3}. On the assumption that a meteor trail behaves like an isothermal plasma, that is, the electron and ion thermal energies are equal, Kaiser (1953) has shown that $\beta = \gamma$. Hence $\beta = \gamma = 10^{-3}$.

The diffusion coefficient for ions has been obtained from studies of the decay rates of meteor trails, as explained by Weiss (1955). These give for D_i, at a height of 100 km, the value 10 m²/s. Hence with $\beta = 10^{-3}$, $D_e = 10^4$ m²/s.

Mobility is difficult to measure directly in a meteor trail. However, from the known behaviour of electrons and ions in gases, Huxley (1952) has shown that $M_e = 1{\cdot}8\times10^6/p$ cm² V^{-1} s^{-1}, where p is the atmospheric pressure in millimetres of mercury. This is stated by Huxley (1952) to be only a crude value for M_e, but it agrees very well with the value obtained from $M_e = D_e.e/kT$ by inserting the values for e (electronic charge), k (Boltzmann's constant), D_e obtained above, and $T = 200$°K. Thus, with $p = 1{\cdot}6\times10^{-4}$ mmHg, $M_e = 10^6$ m² V^{-1} s^{-1}.

A typical length for the system could be that which corresponds to the duration of the radio echo from a meteor trail. A duration of about 0·1 s is quite common, so, in terms of dimensions, $L = (TD_e)^{\frac{1}{2}}$ = about 30 m. Hence $a = 30$ m, and αa^3 = about 1/500 m³.

Equations (9) and (10) have been solved by numerical methods using a high speed computer IBM7090. The space variables are replaced by finite differences and there results a system of ordinary differential equations, since all the variables become functions of τ only. One differential equation appears at each difference interval and describes the behaviour with time of the distribution at that point in space. Thus $\partial n_e/\partial \tau$ becomes $\mathrm{d}n_e(jh)/\mathrm{d}\tau$, where h is the finite difference interval and j is an integer. To simplify the notation $n_e\ (jh)$ is now written $A(J)$. $\partial^2 n_e/\partial \zeta^2$ becomes $[A(J+1)-2A(J)+A(J-1)]/h^2$ and $\partial n_e/\partial \zeta$ becomes $[A(J+1)-A(J-1)]/2h$. Thus equation (9) becomes

$$\frac{\mathrm{d}A(J)}{\mathrm{d}\tau} = [A(J+1)-2A(J)+A(J-1)]/h^2+[A(J+1)-A(J-1)]/2Jh^2$$
$$-A(J)[A(J)-B(J)]/500+\{[A(J+1)-A(J-1)]/1000Jh^2\}.\mathrm{SI}(J), \quad (11)$$

in which $B(J)$ is written for $n_i(jh)$ and $\mathrm{SI}(J)$ is a number calculated for each value of J by evaluating the integral on the right of equation (9) according to Simpson's rule. In the same way (10) becomes

$$10^3\,\frac{\mathrm{d}B(J)}{\mathrm{d}\tau} = [B(J+1)-2B(J)+B(J-1)]/h^2+[B(J+1)-B(J-1)]/2Jh^2$$
$$+B(J)[A(J)-B(J)]/500-\{[B(J+1)-B(J-1)]/1000Jh^2\}\mathrm{SI}(J). \quad (12)$$

Bounds for the calculations are obtained by noting that, at a large distance from the origin, the electron and ion distributions will remain constant in time. This means that the upper limit for J should be fairly large. This limit is taken here to be 20 in order to keep the calculations within practical limits. Hence in (11) and (12) J has the values 1 to 19 so providing 38 differential equations to be integrated simultaneously.

To obtain starting values the initial distributions for electrons and ions are taken to be equal and Gaussian. This is thought to be so for meteor trails (W. G. Elford, personal communication).

Let the initial distributions be of the form $c\exp(-r^2/2G^2)$,
Then the total number of particles in the group is given by

$$c\int_0^R \mathrm{d}r\ r\exp(-r^2/2G^2) = cG^2\{1-\exp(-R^2/2G^2)\}.$$

Writing $cG^2 = N_0$ and choosing R so that $\exp(-R^2/2G^2) = \frac{1}{2}$, half the group lies initially in the region of radius R and the initial distributions are given approximately by

$$\frac{2N_0}{R^2}\exp\left(-\frac{r^2}{R^2}\right).$$

The distance R is taken to be about 2 mean free paths, that is, about $\frac{1}{2}$ m or 1/60 in terms of ζ. The step-length h for the finite difference approximation is then chosen as 1/180, so that there are three steps in the region R. The initial distributions are then calculated in the form

$$A(J) = B(J) = 2N_0\times 60^2\times\exp(-J^2/9); \qquad J = 0,\ 20.$$

The initial values of the integrals $\mathrm{SI}(J)$ can be calculated now in the form:

$$\mathrm{SI}(1) = \tfrac{1}{3}h.h[A(1)-B(1)],$$
$$\mathrm{SI}(L) = \mathrm{SI}(L-1)+\tfrac{1}{3}h.Lh[A(L)-B(L)]+3(L-1)h[A(L-1)-B(L-1)],$$
$$L = 2,4,6,\ \ldots\ ,18,$$
$$\mathrm{SI}(M) = \mathrm{SI}(M-1)+\tfrac{1}{3}h.Mh[A(M)-B(M)]+(M-1)h[A(M-1)-B(M-1)],$$
$$M = 3,5,7,\ \ldots\ ,19.$$

Finally, the initial values of the derivatives $DA(J)$ and $DB(J)$ can be computed, since every term on the right of (11) and (12) is known.

By using the information stored in the computer, the 38 differential equations can be integrated using the Runge-Kutta process. In this, four estimates of the slope are made over a suitable τ step-length and new values of the dependent variables A and B are obtained. This is carried out for each equation in turn and the whole process repeated for as many steps as desired. After a suitable number of integrations, or after each if desired, the computer can be made to print out the new values of A and B.

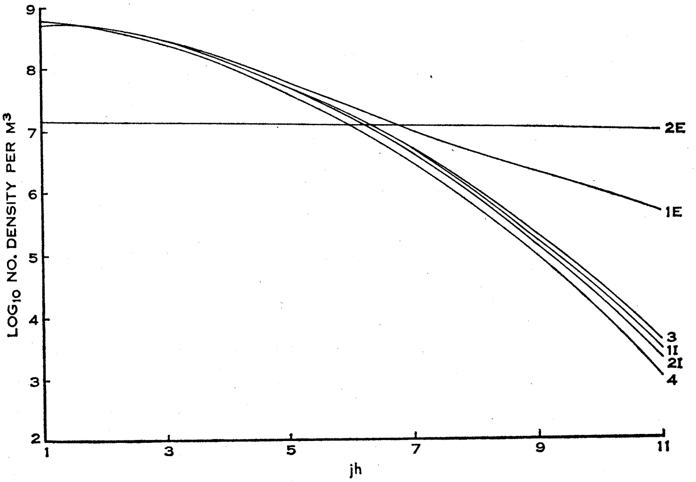

Fig. 1.—Electron (1E) and ion (1I) distributions at $\tau = 3\times10^{-3}$ together with distributions for ambipolar diffusion (3), free electron (2E) and ion (2I) diffusion. The initial distribution for every case is also shown (4).

The choice of the τ step-length is fairly critical to the process. This must be chosen as large as possible in order to economize in machine time. However, if the choice is made too large, the process can become unstable, owing to inaccuracies associated with the finite difference approximations. After some trial and error, the maximum step-length for stability was found to be 10^{-11} when A and B were initially $10^{14}/\mathrm{m}^3$ at the origin. Such a short step-length would require many hours of machine time to produce worth-while results and since this time was not available a step-length of 10^{-6} was chosen for initial concentrations of about $10^9/\mathrm{m}^3$.

IV. Results

With $N_0 = 10^5/\mathrm{m}^3$, so that A and B are about $10^9/\mathrm{m}^3$ at the origin, and a τ step-length of 10^{-6}, the integrations have been carried out over 3000 steps and the resulting values of A and B plotted against jh. The equations for electron and ion self-diffusion—in the absence of electrostatic forces—and for ambipolar diffusion have been integrated in exactly the same way.

These results are all shown in Figure 1 and indicate that for densities greater than about $10^7/m^3$, electrons and ions diffuse together at the ambipolar diffusion rate.

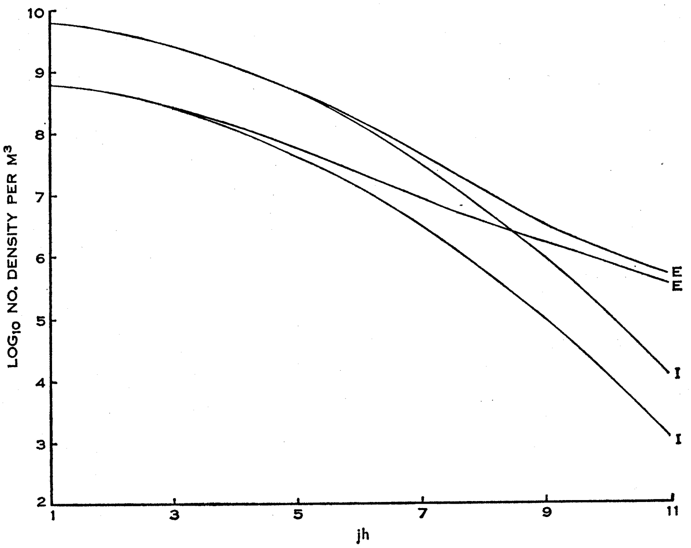

Fig. 2.—Electron (E) and ion (I) distributions at $\tau = 2 \times 10^{-4}$ showing the effect of a larger ($\times 10$) initial distribution.

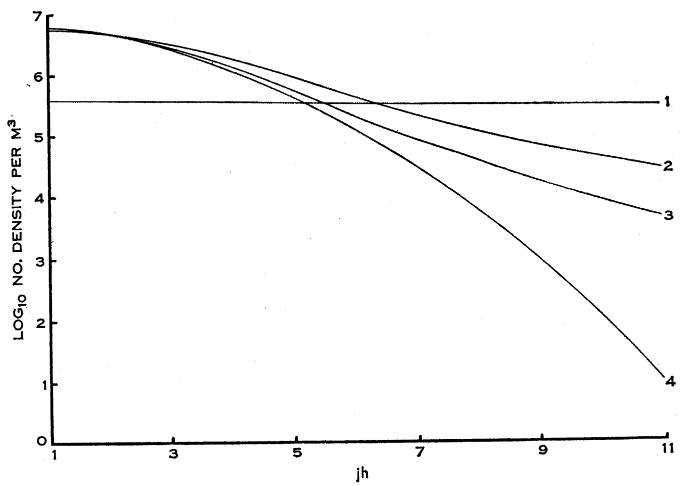

Fig. 3.—Electron distribution at $\tau = 10^{-3}$ showing (1) free electron diffusion, (2) distribution for $\alpha a^3 = 1/55$, and (3) distribution for $\alpha a^3 = 1/11$. (4) shows the initial distribution.

At lower densities the charges tend to separate, indicating that the electrostatic forces are no longer powerful enough to inhibit differential diffusion.

It can be inferred that for an initial concentration of $10^{14}/\mathrm{m}^3$, which is more typical of a low density meteor trail, the charges will spread out at the ambipolar diffusion rate for the duration of a radio echo. This inference has been tested to a small extent by increasing N_0 by 10 times and repeating the computations with a shorter step-length. The results (Fig. 2) show that the effect of the increase is to keep the electron and ion concentrations more nearly the same further away from the origin. Separation still occurs at about the same density as before.

V. Discussion

The transition from free to ambipolar diffusion has been discussed by Allis and Rose (1954), who showed that the transition occurs over many orders of electron density. This is consistent with the present results. Allis and Rose have used the value 32 for the ratio M_e/M_i (for hydrogen), whereas this ratio is taken here to be 10^3. However, the results are not very sensitive to the value a assigned to this parameter. For example, by taking $M_e/M_i = 500$ and repeating the calculations very little difference was found from the distributions for $M_e/M_i = 1000$.

The parameter αa^3 does have a marked effect on the results. This appears as a factor in the terms of equations (9) and (10) which arise from electrostatic forces and is essentially the ratio M_e/D_e. The effect of increasing the size of this ratio is to delay the transition from ambipolar to free diffusion, so that the transition occurs at lower density as the ratio is increased. Electron distributions for $\alpha a^3 = 1/55\ \mathrm{m}^3$ and $\alpha a^3 = 1/11\ \mathrm{m}^3$ are shown on Figure 3.

VI. Acknowledgment

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VII. References

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