PROPERTIES OF THE LUNAR SURFACE AS REVEALED BY THERMAL RADIATION

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Summary

Thermal radiation from the moon has been previously measured by both optical and microwave techniques, and this has led several workers to estimate the thermal properties of the lunar surface. Using the assumption that the thermal properties are proportional to the temperature, the uniform surface corresponding to the observed optical values is calculated. Possible mixed surfaces are also evaluated. These are examined to estimate the likely variation in microwave radiation, and by comparison with observed results it is shown that the most probable surface consists partly of rock or gravel overlain by a thin layer of fine dust, and partly of areas with dust extending to beyond the depth from which the microwave radiation emanates.

I. Introduction

Radiation received from the lunar surface has been measured by numerous workers using optical means during both a lunation and an eclipse and radio waves during a lunation. Epstein (1929), Jaeger and Harper (1950), Jaeger (1953), and others have attempted to find the necessary thermal properties which, if they applied to the lunar surface, would give temperatures equal to those observed. All workers endeavoured to find the properties of an "average" surface, i.e. assuming that the surface is thermally uniform and that:

(i) there is insignificant heat flow from the deep layers of the moon;
(ii) the moon radiates as a black body;
(iii) the thermal properties are independent of temperature.

On this basis Jaeger (1953), whose work is by far the most exhaustive, concluded "that it is difficult to fit both the optical and microwave results with any model" because of "the rather high mean temperature demanded by the microwave results and the rather low temperature during the lunar night".

Jaeger and Harper (1950) looked at the resultant effect if assumption (iii) was incorrect, but no experimental evidence was available until work by Scott (1957) showed that the thermal conductivity of -80 mesh perlite *in vacuo* was approximately proportional to the absolute temperature. Specific heat values for materials such as Fe_3O_4, Fe_2S, $CaCO_3$, and Al_2SiO_5 listed in the International Critical Tables suggested that a similar relation might apply between the specific heat and the temperature. Assuming that these relations were accurate over the temperature range of interest, Muncey (1958) showed that this would resolve the dilemma arising from the difference in mean temperatures derived by the two methods. This paper extends the study, assuming a linear relation between the thermal properties and the absolute

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temperature, to the varying temperatures during a lunation and an eclipse. Again, a better agreement with observed results is achieved than heretofore; the status of the linear relation is still that of an assumption which, for the moment, appears to be reasonable for some terrestrial materials and provides better agreement with lunar results.

With this assumption, an "average" surface that would fit the observed optical radiation values exactly is evaluated. The paper discusses variations of this result, including "mixtures" of two types of surfaces which would give radiation results within the experimental range of the optical observations, and examines which of these would also give rise to the variations in radiation as observed by microwave studies. Material whose thermal inertia $(K\rho C)^{\frac{1}{2}}$—in c.g.s. units and the symbolism of Section III—is of the order of 1/1000 is referred to as dust, and that of the order of 1/10 to 1/500 is referred to as rock or gravel, following the nomenclature of Jaeger (1953). It is assumed that both for dust and for rock or gravel the thermal properties and temperature are linearly related. This assumption is desirable for analytical convenience and even if incorrect in the case of rock, this is unlikely to be serious.

II. Experimental Data

Observations of lunar radiation have been made by the Earl of Rosse (1870), Pettit and Nicholson (1930) Pettit (1940), Piddington and Minnett (1949), Sinton (1955, 1956), Akabane (1955), Gibson (1958), and Geoffrion, Korner, and Sinton (1960). For the purposes of this study, the variation in the radiation from a given point with time is required; among the results that satisfy this requirement and appear to be mutually consistent (e.g. temperatures in excess of 400 °K appear most unlikely), only those studied by Jaeger appear to be acceptable. Geoffrion, Korner, and Sinton give temperatures of the central area of the moon at times when the temperatures are changing rapidly. Their results agree with those of the present calculation but do not provide any additional evidence from which to judge which particular thermal model appears more probable.

The results have been used without probing the effect of assumptions made by previous workers. The data have been assumed to apply to a central area of the moon (of the order of 100 km square) and are, in essence:

1. Solar radiation absorbed at lunar noon: $1\cdot71$ cal cm^{-2} min^{-1}. Pettit and Nicholson (1930) derive this figure from solar observations and lunar reflectivity.
2. Surface temperature at lunar midnight: 120 °K. Pettit and Nicholson (1930) measured this temperature near the eastern limb. This is the only published midnight temperature, but Sinton (personal communication) has remarked that a similar temperature applies in the central area.
3. Eclipse data:
   Period of penumbra 280 min
   Period of umbra 140 min
   Radiation from the surface
      at the beginning of umbra $0\cdot12$ cal cm^{-2} min^{-1}
      at the end of umbra $0\cdot08$ cal cm^{-2} min^{-1}.

Pettit (1940) reports these results from an eclipse on October 27, 1939 relating to an area 0·17 radii from the centre.

4. Central temperature derived from microwave measurements:

$$249+52 \cos(\Omega t-\tfrac{1}{4}\pi)\ ^\circ\mathrm{K}.$$

Piddington and Minnett (1949) give this figure from observations of the whole disk and calculation of the central temperature. Their methods have been questioned (see Jaeger 1953) but only the phase lag is directly involved in the analysis below and this follows directly from the experimental data.

III. Calculations

Let the following notation be adopted:

the temperature at any point	v
the temperature at any point, squared	V
the thermal conductivity	$K = kv$
the specific heat	$C = cv$
the density	ρ
the time	t
the distance (into the lunar surface)	x
the absorbed radiation	R
the emitted radiation	σv^4.

The normal relations of temperature, distance and time apply.

In the solid,

$$C\rho \frac{\partial v}{\partial t} = \frac{\partial}{\partial x} K \frac{\partial v}{\partial x}, \tag{1}$$

and, at the surface,

$$\sigma v^4 - K\frac{\partial v}{\partial x} = R. \tag{2}$$

Writing V for v^2, these can be seen to be

$$\frac{\partial V}{\delta t} = \frac{k}{c\rho}\,\frac{\partial^2 V}{\partial x^2}, \tag{3}$$

and

$$\sigma V^2 - \tfrac{1}{2}k\frac{\partial V}{\partial x} = R. \tag{4}$$

Equation (3) is of the same form as the normal diffusion equation for heat flow in a solid (equation (1)) and hence theory developed previously may be applied, due regard being paid to the difference in meaning of the symbols. In particular, the treatment of the steady periodic state (wherein V may be written as a sin bt) as developed by van Gorcum (1950) and extended by Muncey (1953) can be applied (see also Carslaw and Jaeger 1959, para. 3.7). Across an infinite slab having temperatures v_a and v_b and heat flows f_a and f_b on the two faces, two linear relations apply which in matrix form are:

$$\begin{vmatrix} v_b \\ f_b \end{vmatrix} = \begin{vmatrix} A & B \\ C & D \end{vmatrix} \cdot \begin{vmatrix} v_a \\ f_a \end{vmatrix}, \tag{5}$$

wherein A, B, C, and D depend on the thermal properties of the material of the slab, which need not necessarily be homogeneous.

At depth in the lunar surface the (sinusoidal) value of the "heat flow" f_b is zero and one of the two equations (5) may be written:

$$Cv_a + Df_a = 0,$$

or

$$v_a/f_a = -D/C. \tag{6}$$

Let D/C be called the parameter ratio. It will, in general, be complex and will vary with frequency.

If a cycle (represented by n ordinates) for σV^2 be assumed corresponding to a lunation, V may be calculated at each time, a harmonic analysis made for the components of V, and from these and the parameter ratios of the surface, the "heat flow" at each frequency may be found. Harmonic synthesis coupled with equation (4) may then be used to obtain a new trial value for σV^2, the emitted radiation. Continued iteration will determine the radiation cycle corresponding to the particular set of parameter ratios derived from the assumed surface. It is convenient to examine the eclipse case by the same method, choosing a cycle period somewhat longer than the penumbra period. This model is equivalent to an infinite succession of "eclipses". A steady-state flow will need to be included to compensate for the fact that the average "irradiation" will be too small, and the value of this term and of the total period will have to be chosen so that the value of V is steady and equal to that at lunar noon just prior to the start of each "eclipse". It was found convenient to choose a period of 425·24 min, i.e. 0·01 of the lunation period, and a satisfactorily constant value of emitted radiation prior to penumbra was obtained.

IV. Results

(a) *Optical Observations*

Throughout the calculations, the cycles of V were represented by 64 ordinates and all work was carried through with 32 harmonics. It was found by trial that the only uniform surface that would satisfy both the lunation and eclipse results consisted of a dust layer of resistance 180 000 cm^2 sec degK cal^{-1} and capacity 0·085 cal cm^{-2} degK^{-1} (both at 350 °K) overlaying a gravel of thermal inertia $(K\rho C)^{\frac{1}{2}}$ of 1/400, also at 350 °K. The full lunation temperature cycle and the eclipse cycle are shown in Figures 1(a) and 1(b), with experimental results for comparison.

It was found that the values of the calculated ordinates at which comparison was made were largely determined by the parameter ratio relevant to the lunation frequency (the real part only), and to a frequency 100 times larger and relevant to the eclipse values. This is not surprising when the importance of the first harmonic is realized, and also that the parameter ratio varies smoothly with frequency. The values of these parameter ratios form the axes of Figure 2, and the region within the dotted line shows the approximate values necessary so that temperature cycles derived therefrom will agree with observations within the experimental error. (Midnight temperature ±5 degK, radiation during umbra ±0·005 cal cm^{-2} min^{-1}).

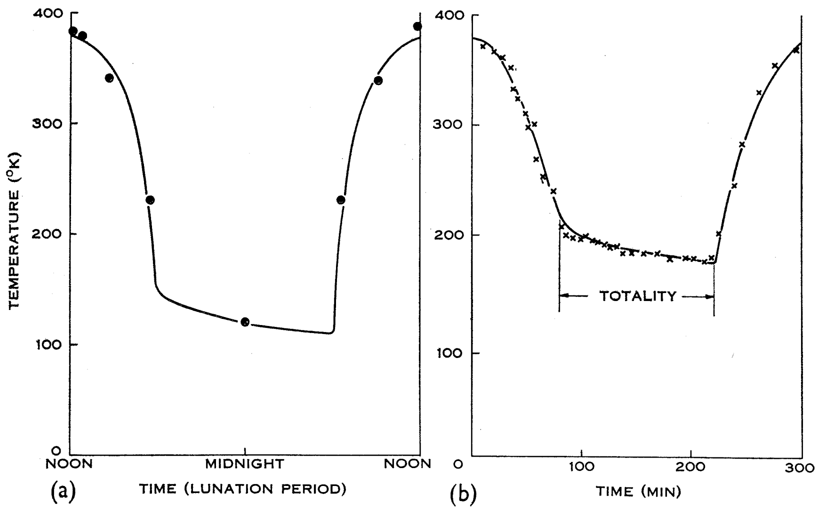

Fig. 1.—Lunar temperature cycles. (*a*) Lunation, (*b*) eclipse.
⊙ After Geoffrion, Korner, and Sinton (1960); ⊗ after Pettit and Nicholson (1930); × after Pettit (1940).

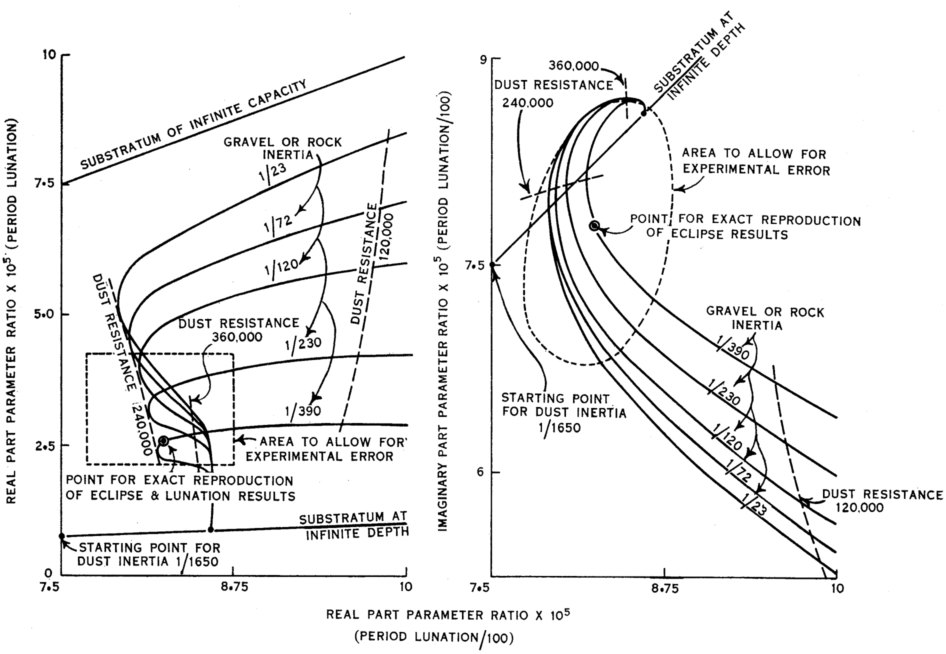

Fig. 2.—Relation between parameter ratio and thermal properties of the lunar surface at 350 °K (dust inertia 1/1450).

The figures further show, for a surface film of one particular thermal inertia, $(K\rho C)^{\frac{1}{2}}$ of 1/1450 at 350 °K, the corresponding thermal resistance of the surface and the inertia of the underlying material for various values of the parameter ratio. The curves for slightly different surface values of thermal inertia will be of comparable shape, the point relating to very large values of surface resistance being indicated.

Little variation in the inertia of the surface dust seems possible, because of the resultant effect on the eclipse results, but the properties of the underlying gravel or rock may vary considerably, particularly if it be postulated that the strata underlie only part of the area and that elsewhere the dust is of considerable depth. For given values of the inertia of the rock or gravel substratum, estimates were made of (1) the proportion covered by a deep layer of dust and (2) the resistance of the dust layer over the substratum. These are presented in Figure 3.

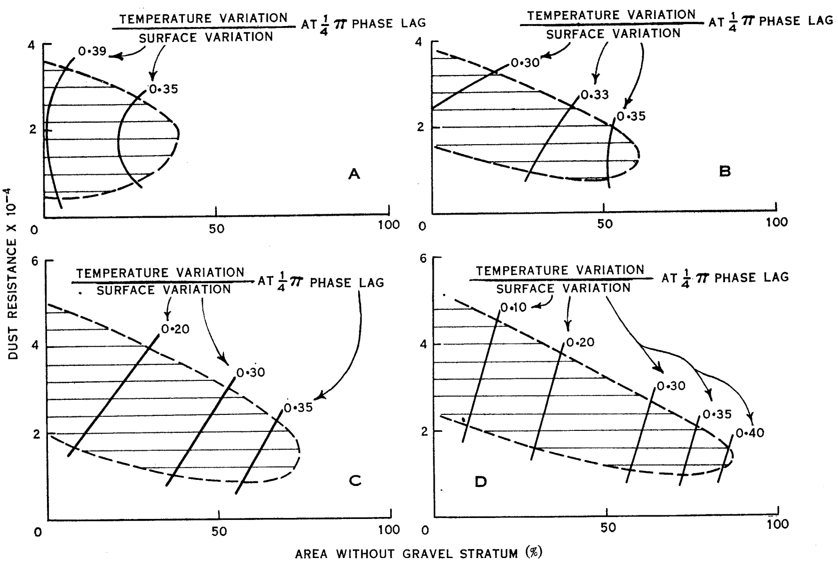

Fig. 3.—Possible models of lunar surface satisfying optical results (dust inertia 1/1450). Subsurface temperature variation observed by microwave methods. Temperature variation/surface variation = 0·37. Gravel or rock inertia: *A*, 1/390; *B*, 1/230; *C*, 1/72; *D*, 1/23.

It will be observed that no surface model of constant thermal inertia will satisfy the optical results. A uniform surface of inertia $(K\rho C)^{\frac{1}{2}}$ of 1/1450 to 1/1600 at 350 °K would be in reasonable agreement with the eclipse results, but such a surface would fall to 100 °K or below at lunar midnight. The experimental results suggest than the actual surface is not likely to be as cold as this.

(*b*) *Microwave Results*

Thermal radiation of long wavelength emanates from beneath the lunar surface, the emitted value corresponding to a temperature resulting from an "integration" of the variation with depth and an absorption by the material above successive layers. The radiation is observed to lag one-eighth of a lunation cycle at the fundamental frequency and to have a temperature variation of 52 degK at the fundamental frequency. This corresponds to a change in V ($= v^2$) of $0 \cdot 37$ times that at the surface.

The method of Muncey (1953) enables the variation V at the dust-gravel "interface" to be calculated for any particular surface model, and by standard heat transfer methods (e.g. Carslaw and Jaeger 1959, para. 2.6) the value of V for any depth can be calculated. The ratio of the variation of V at depth (magnitude and phase) compared with that at the surface can therefore be found, and from this the magnitude ratio at a phase lag of one-eighth of a lunation can be derived. For mixed surfaces it is appropriate to take the average (weighted for area) of the values of V at points where there is an equal thermal capacity "above" them. This calculation was performed for many of the cases shown in Figure 3, on which the resulting ratios are indicated.

V. Discussion

The observed behaviour of lunar temperatures during a lunation and an eclipse (assuming that the thermal capacity and conductivity vary with temperature) can be reproduced very closely. Various models can be found that will duplicate the lunar behaviour assuming a uniform surface; in all cases they consist of dust of inertia $(K\rho C)^{\frac{1}{2}}$ of about 1/1500 at 350 °K overlying rock or gravel. It is also possible to find models consisting partly of such surfaces and partly of surfaces exclusively of dust extending beyond the depth that influences the thermal radiation. The latter agree with popular conceptions. If the simplification is accepted that the microwave radiation arises from a particular layer beneath the surface, the model with large dust areas appears the more likely. With a rock substratum having $(K\rho C)^{\frac{1}{2}}$ of 1/20 at 350 °K, up to 80% of the total area may be covered with deep layers of dust, and only with a very low inertia substratum can a uniform surface be postulated.

It does not seem possible that greater precision in the data available at present would resolve the remaining doubt, although confirmation of midnight temperatures would be desirable. Following studies to confirm the reasonableness of the assumption relating thermal properties and temperature, the most useful additional data would be microwave measurements at other frequencies (both higher and lower) to provide information at other depths, and eclipse observations at long wavelength (e.g. Sinton 1956).

VI. References

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