

AN INVESTIGATION ON THE THERMAL CONDUCTIVITY OF POROUS MATERIALS AND ITS APPLICATION TO POROUS ROCK

By A. SUGAWARA* and Y. YOSHIZAWA*

[*Manuscript received March 27, 1961*]

Summary

The thermal conductivity of porous materials, particularly the effect of porosity and of temperature on thermal conductivity, has been investigated. A "comparison method" was used for measurement and the experiment was carried out with three types of models. A generalized formula for thermal conductivity is derived empirically. This formula can represent other formulas on the thermal conductivity of porous materials, and the formula can be applied to sandstones having different porosity as a typical example of porous materials.

I. INTRODUCTION

Owing to recent developments in science and engineering, such as geophysics, soil science, and mechanical and chemical engineering, the study of the thermal conductivity of porous materials is becoming increasingly important. The amount of pores in porous materials plays an important role among the many factors which influence thermal conductivity. However, no formulas are known that represent reasonably the relationship between thermal conductivity and porosity. Therefore correct formulation of the effect of porosity on thermal conductivity is of great importance.

This effect has been studied extensively, and numerous attempts (Euchen 1932; Russell 1935; Ribaud 1937; Waddams 1944; Franc and Kingery 1954; Loeb 1954; Krischer and Esdorn 1956; Kingery and Klein 1958; Somerton 1958) have been made to derive simplified relationships by approximating the complex effect of pore space. It has been known that not only the porosity but also the size, the shape, the orientation of the pore, and the emissivity in the pore have considerable effects on thermal conductivity. Even if these effects were known separately, it is impossible to analyse theoretically these effects because of three-dimensional irregularity of the above factors. Eventually therefore, all the formulas on thermal conductivity influenced by porosity had to be empirical and expressed as a function of porosity alone (Euchen 1932; Russell 1935; Ribaud 1937; Franc and Kingery 1954; Krischer and Esdorn 1956) with a shortcoming of not representing satisfactorily the actual state of thermal conductivity.

In the present paper a rather more reasonable formula relating thermal conductivity than any of those previously published is obtained from the results of fundamental experiments. By varying the empirical exponent " n " in this formula, the actual overall thermal conductivity can be represented more correctly than by any other formula.

* Department of Mining, Faculty of Engineering, Tohoku University, Sendai, Japan.

II. METHOD

In the present work the comparison method by linear heat flow has been adopted to measure the thermal conductivity. A sample material and a standard material, both in the form of plates, are brought face to face in close contact and a steady heat flow is sent through the plates, establishing a steady linear flow through the plate centres. From the measured values of the temperature gradients in the two plates and from the known thermal conductivity of the standard material, the thermal conductivity of the sample material is obtained.

The comparison method has an inherent defect as applied to the relatively thick sample; lateral heat loss to the surroundings reduces the accuracy of measurement. To limit the error thus caused to 1%, the thickness is required to satisfy the condition given by Tanazawa's (1932) theory that the diameters of the standard and sample plates should be more than five times their combined thickness.

Since in the steady state the temperature gradients in the two plates are obtained in the forms of $(\Delta T/d)$ and $(\Delta T_0/d_0)$, the thermal conductivity of the sample is given by

$$\lambda = \lambda_0 \left(\frac{\Delta T_0}{d_0} \right) \left(\frac{d}{\Delta T} \right), \quad (1)$$

where λ, λ_0 = thermal conductivities of sample and standard plates respectively, in kcal/m hr degC,

$\Delta T, \Delta T_0$ = temperature differences between the two faces of the sample and standard plates respectively, in degC,

d, d_0 = thickness of sample and standard plates respectively, in mm.

A diagram of the experimental apparatus has been published and explained in detail by Sugawara (1961).

III. SAMPLE PREPARATION

Each sample was made to conform to the condition required by Tanazawa's theory; the shape was a flat circular disk with surfaces finished as smooth as possible. Attention was paid to having the thickness constant at every point.

The samples used were as follows.

(a) An Aggregation of Glass Balls having Small Diameter

An aggregation of small glass balls is considered most ideal as the model of porous material and very convenient to explain actual phenomena, but in the natural piled up state as shown in Figure 1 (a), called the cubic type, the porosity is 26.0% regardless of ball diameter. So, in order to obtain a different porosity, balls were linked together with very fine threads of the same glass as the ball, as shown in Figure 1 (b). The porosity was varied by changing the distance between the balls.

The diameter of the glass balls used was 3.1 mm, and the porosity was 26.0% for the natural piled up state and was 80.0% for the linked balls.

The most serious drawback in this experiment is the uncertainty of the conductivity of the solid constituent of the material as a basis for calculation.

Many cases are known in which the method of determining such conductivity is that a more or less straight line relationship between the overall thermal conductivity and the porosity is assumed and, by the use of a few samples of

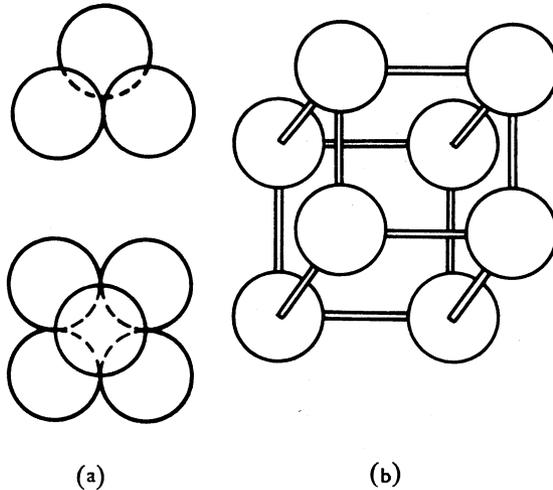


Fig. 1.—Glass ball samples.

different porosity and by extrapolation to zero volume fraction, the required conductivity is obtained. In the present case, the glass balls were melted down and glass plates were cast with which the conductivity of the glass was determined.

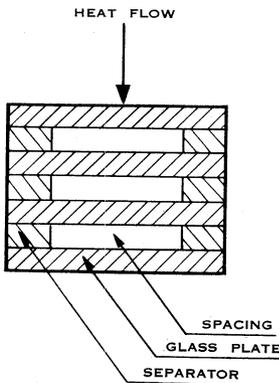


Fig. 2.—Multilayered glass sample.

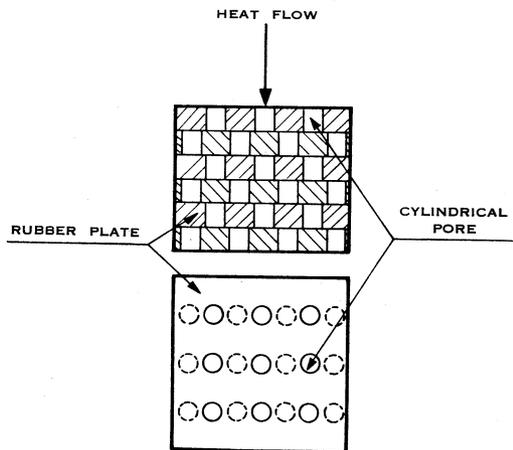


Fig. 3.—Pored rubber plate sample.

(b) Multilayered Glass Plates

Krischer and Esdorn (1956) showed that, in measuring thermal conductivity, porous materials can be simulated by multilayered plates of glass. This idea was followed up and a set of multilayered glass (as shown in Fig. 2) was prepared.

Several glass plates were placed face-parallel, the spacing between plates was kept constant by means of a separator fixed around the edges, the ratio of the total sum of the spacing to the total volume of the glass array representing the porosity of the array.

The porosities used in this experiment were 25.0, 50.0, and 75.0%.

(c) *Cylindrical-pored Rubber Plates*

A piled set of flat and smooth rubber plates was prepared as a sample. Cylindrical pores 2.0 mm in diameter were drilled through every plate with a given pitch regularly over its whole surface. In piling the plates, every other plate is shifted half the pitch so that all the pores are closed off by adjacent plates as shown in Figure 3.

For varying the porosity, the pitch between the pores was changed. The porosities used were 12.5, 26.0, and 50.0%.

IV. RESULTS

Experiments were made on the above three types of samples under given porosities. The fluids with which the pores were filled were dry air and distilled water. Thermal conductivity was obtained as a function of temperature with porosity as parameter.

From the results of the measurement, thermal conductivity (λ) versus temperature (t) diagrams and thermal conductivity (λ) versus porosity (ψ) diagrams were drawn.

(a) λ versus t Diagrams

λ versus t diagrams obtained on the glass ball, the multilayered glass, and the pored rubber plates are shown in Figures 4, 5, and 6 respectively with porosity as parameter.

In Figures 4 and 5

$$\lambda_{sg} > \lambda_w > \lambda_a,$$

in Figure 6

$$\lambda_w > \lambda_{sr} > \lambda_a,$$

where suffixes are a , air; sg , glass; sr , rubber; w , water.

In each case the relationship between (λ) and (t) is linear. In Figures 4 and 5, the lines placed above the line (λ_w) and in Figure 6, the lines above the line (λ_{sr}) show the case when water was used as the filling fluid. The lines below them relate to when air was used.

(b) λ versus ψ Diagrams

λ versus ψ diagrams with temperature as parameter can be obtained by tracing λ versus t diagrams vertically. λ versus ψ curves for the glass ball, the multilayered glass, and the pored rubber plate samples are shown in Figures 7, 8, and 9 respectively. When the thermal conductivity of the solid constituent is greater than that of the fluid in the pores, the λ versus ψ curves are convex against the ψ axes as shown in Figures 7 and 8 and when otherwise, λ versus ψ curves are concave as shown in the case of water saturation of Figure 9.

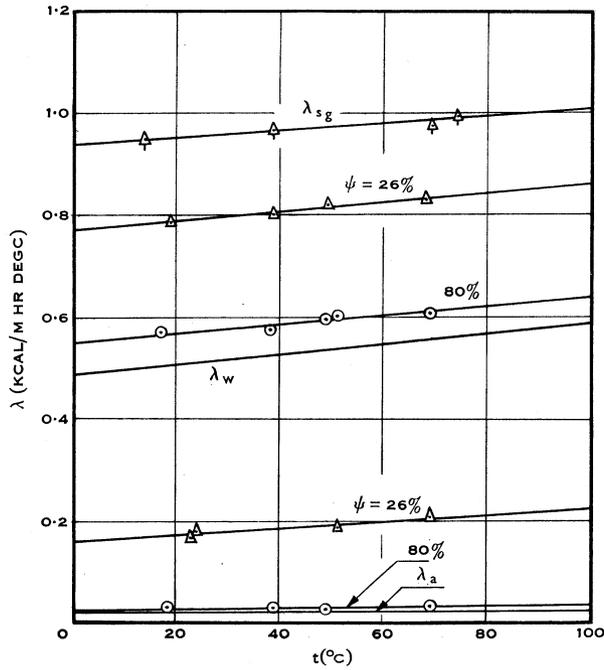


Fig. 4.— λ versus t diagram obtained on glass ball samples.

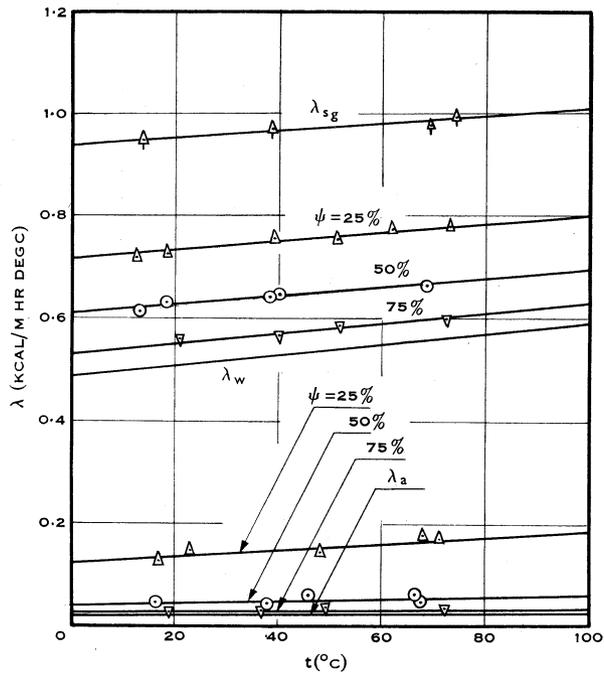


Fig. 5.— λ versus t diagram obtained on multilayered glass sample.

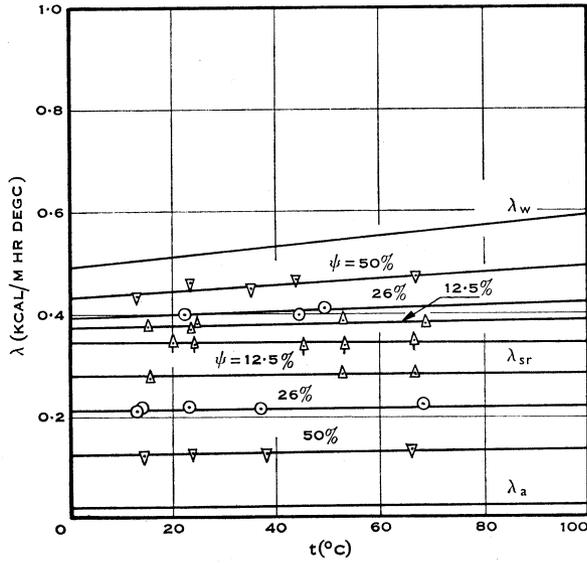


Fig. 6.— λ versus t diagram obtained on pored rubber plate sample.

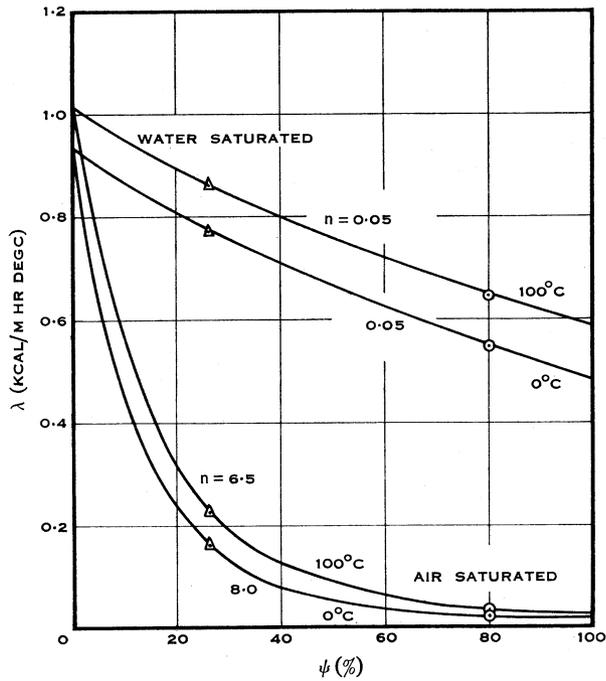


Fig. 7.— λ versus ψ diagram obtained on glass ball samples.

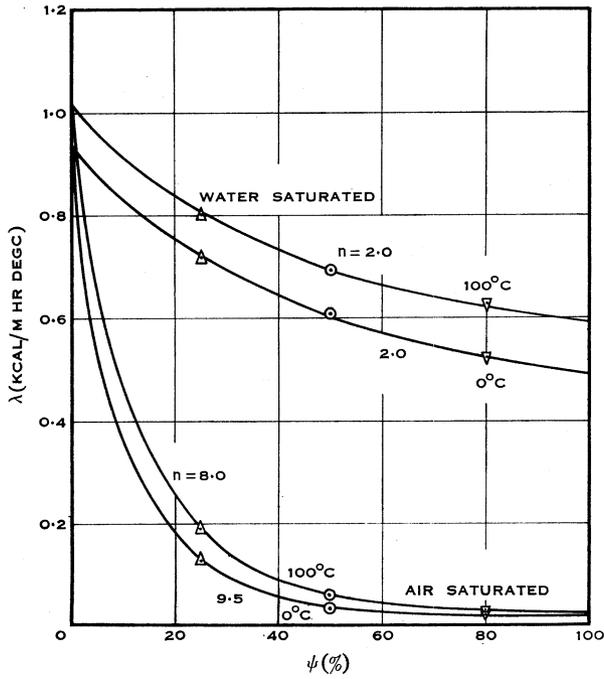


Fig. 8.— λ versus ψ diagram obtained on multilayered glass sample.

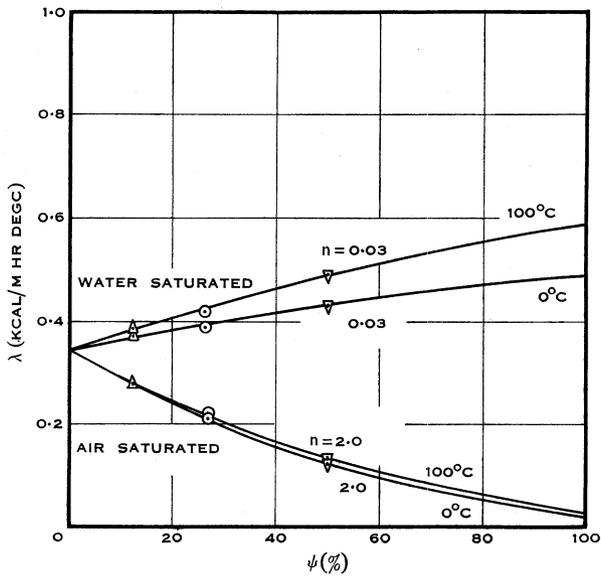


Fig. 9.— λ versus ψ diagram obtained on pored rubber plate sample.

Generally, it can be seen that the porosity and the fluid that fills the pores influence the thermal conductivity very much, the porosity in particular when the difference between the conductivity of solid constituent and that of the fluid is very great.

From these diagrams, the following general formula is derived,

$$\lambda = (1 - A)\lambda_s + A\lambda_f \quad \text{kcal/m hr degC,} \quad (2)$$

where $A = \frac{2^n}{2^n - 1} \left\{ 1 - \frac{1}{(1 + \psi)^n} \right\}$,

λ = overall thermal conductivity of porous materials,

λ_f, λ_s = thermal conductivity of fluid and solid constituent respectively,

ψ = porosity,

n = empirical exponent determined by mode of packing, pore size, pore shape, and emissivity inside of the pore, and $n > 0$.

The values of n for each λ versus ψ curve are shown in Figures 7, 8, and 9.

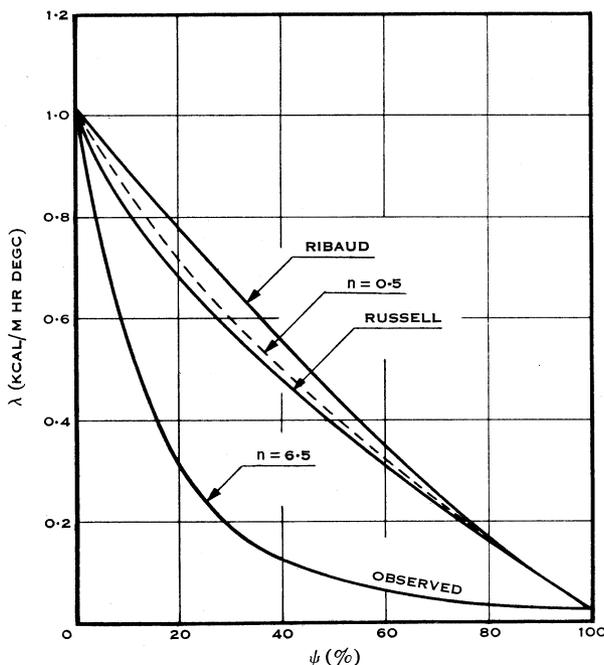


Fig. 10.— λ versus ψ diagram compared with other results.

V. COMPARISON WITH OTHER RESULTS

There are many formulas concerning the relationship between thermal conductivity and porosity; representative are those of Eucken (1932), Russell (1935), Ribaud (1937), and Loeb (1954).

By Ribaud's and Russell's formulas applied to the case of the air-filled glass ball specimen and by the use of the values of λ_s and λ_a at 100 °C mentioned above, λ versus ψ curves were obtained by calculation and are shown in Figure 10,

on which the curve obtained in the present experiment is given, marked "Observed", for comparison.

The very great difference between these curves can probably be attributed to the fact that all those authors referred to above treated the thermal conductivity as a function of porosity alone and ignored the influence of pore size, pore shape, material structure, etc.

Ribaudo's and Russell's formulas are as follows :

$$\lambda = \lambda_a \psi^3 + \lambda_s (1 - \psi^3), \quad \text{Ribaudo}$$

$$\lambda = \lambda_s \frac{\psi^3 + Q(1 - \psi^3)}{\psi^3 - \psi + Q(1 - \psi^3 + \psi)}, \quad \text{Russell}$$

where $Q = \lambda_s / \lambda_a$.

Other formulas have similar tendencies to the above two in deviating from the observed curve shown in Figure 10.

Allowing a small percentage of error, if we represent the two calculated curves of Figure 10 by a curve shown by the dotted line in the figure, the value of n of this curve becomes 0.5. So the formula (2) can cover any other formula on the thermal conductivity of porous material.

VI. APPLICATION TO POROUS ROCK

For applying formula (2) to practical cases, we choose sandstones having different porosities as a typical example of porous material. The sandstone is sedimentary rock which consists of aggregations of small particles. The sandstone used, called "Nanaura", was obtained from the Yotsuyama Colliery, Mitsui Mining Co., located in Fukuoka Prefecture.

The characteristics of the sandstones by visual observation were as follows :

grain size ; medium (diameter $\frac{1}{2} \sim \frac{1}{4}$ mm by U.S.
Bureau of Soils Classification)
matrix ; calcareous
sorting ; well and uniform
colour ; grey and light grey.

Measurements were carried out for two cases on each sample, one was dry (did not contain moisture) and the other was water-saturated. In order to obtain the dry sample, it was dried for 2 hr in an air oven at 110 °C and was put into a closed vessel in which the measurement was carried out. On the other hand, in order to saturate the pore space with water, the sample was supported as shown in Figure 11 ; one face of the flat sample is in air and the other is in water. In this case, water was introduced into the pore space by capillary pressure through the pores and the air in the pores was driven out from the open face. Thus we obtained a fully water-saturated sample.

On measuring the thermal conductivity of the water-saturated sample, to prevent any convection effect caused by water movement (sometimes moisture movement) due to a temperature gradient in the sample, the temperature difference between the two faces of the sample plate was kept as small as possible during the measurement.

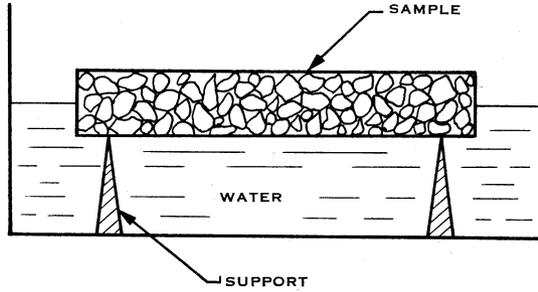


Fig. 11.—Cross section of diagram for saturating the pore spaces of sandstone specimen with water.

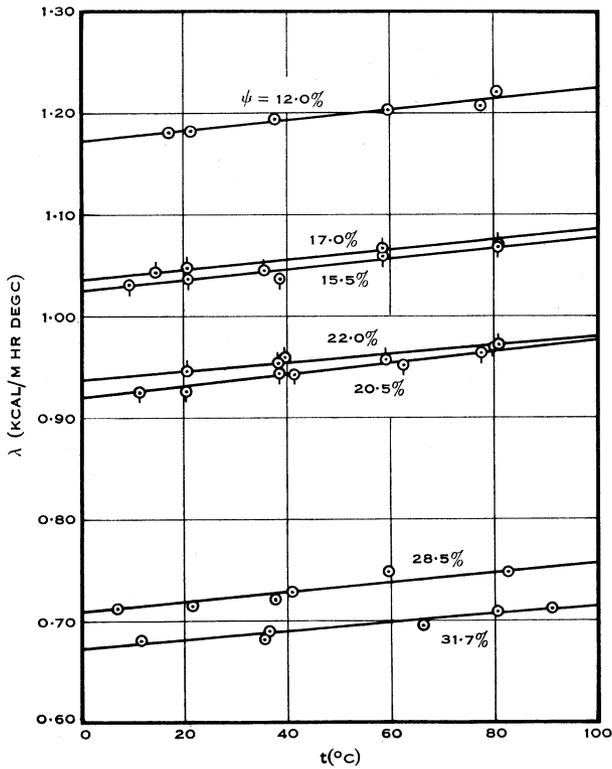


Fig. 12.— λ versus t diagram obtained on sandstones saturated with air.

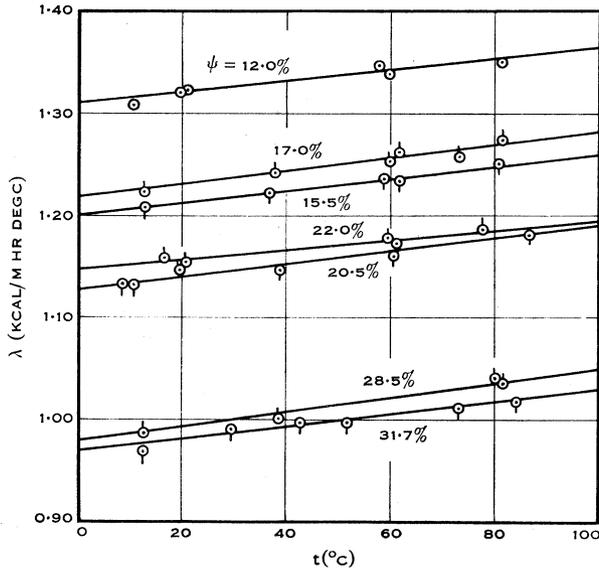


Fig. 13.— λ versus t diagram obtained on sandstones saturated with water.

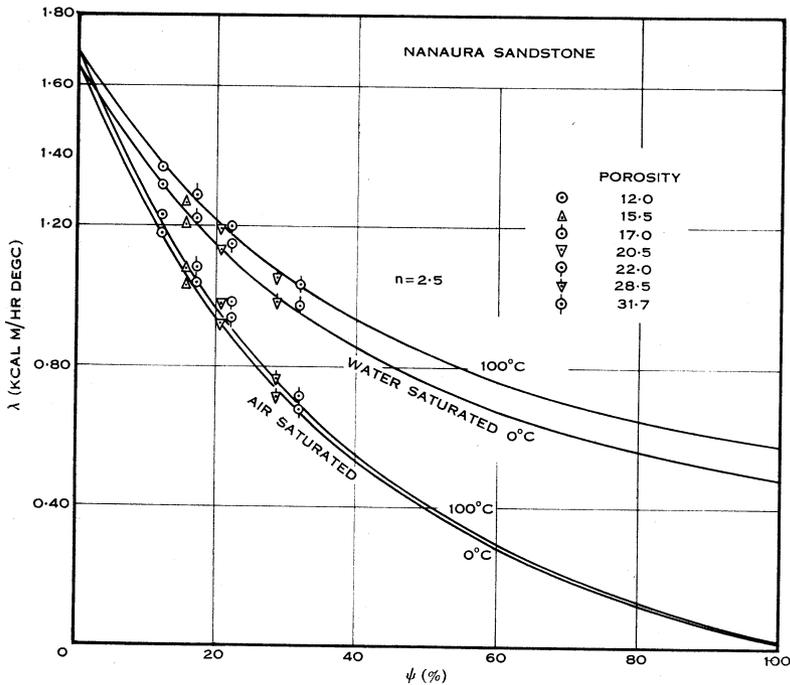


Fig. 14.— λ versus ψ diagram obtained on sandstones.

Results of measurements are shown in Figures 12 and 13. From these two λ versus t diagrams with porosity as parameter it can be recognized that the relationship between thermal conductivity and temperature is linear also; thermal conductivity increases with rise in temperature.

The λ versus ψ diagram with temperature as parameter can be drawn by tracing λ versus t diagrams vertically, and it is shown in Figure 14. Thus, applying formula (2) to these four curves, the value of n becomes 2.5.

This fact shows that formula (2) is in complete agreement with the experimental results and that this formula can apply immediately to practical examples.

VII. CONCLUSION

The effect of porosity on thermal conductivity is confirmed experimentally for porous materials and following facts are shown.

(1) If the thermal conductivity of the solid constituent is larger than that of the fluid in the pores, the thermal conductivity of a porous material is decreased with porosity increment.

(2) If the thermal conductivity of the solid constituent is smaller than that of the fluid, the thermal conductivity of a porous material is increased with porosity increment.

The generalized formula (2), with which above two cases can be covered simply by varying the value of n , has been obtained empirically.

This formula can represent any other formula on thermal conductivity of porous material by varying n as stated in Section V. We show that this formula is in complete agreement with the experimental results.

VIII. REFERENCES

- EUCKEN, A. (1932).—*V.D.I. Forschungsheft* **3** (353): 16.
FRANCL, J., and KINGERY, W. D. (1954).—*J. Amer. Ceram. Soc.* **37**: 99–107.
KINGERY, W. D., and KLEIN, L. (1958).—*Trans. Amer. Soc. Mech. Engrs.* **80**: 705–10.
KRISCHER, O., and ESDORN, H. (1956).—*Forsch. IngWes.* **22**: 1–8.
LOEB, A. L. (1954).—*J. Amer. Ceram. Soc.* **37**: 96–9.
RIBAUD, M. (1937).—*Chal. et Industr.* **18**: 36–43.
RUSSELL, H. W. (1935).—*J. Amer. Ceram. Soc.* **18**: 1–5.
SOMERTON, W. H. (1958).—*J. Petrol. Tech.* **10** (5): 61–4.
SUGAWARA, A. (1961).—*J. Appl. Phys. Japan* **30**: 17–23.
TANAZAWA, Y. (1932).—*J. Japan Soc. Mech, Engrs.* **35**: 325–32.
WADDAMS, A. L. (1944).—*J. Soc. Chem. Ind., Lond.* **63**: 357.