THE TOWNSEND IONIZATION COEFFICIENTS IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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Summary

An expression is obtained for the first Townsend ionization coefficient in uniform crossed electric and magnetic fields, and shown to be in better agreement with observation than previous theoretical expressions. The "equivalent pressure" concept for the effect of a transverse magnetic field on this coefficient is shown to be a valid approach to the problem, although the value for the equivalent pressure obtained in this analysis differs from the values given by earlier authors.

The effect of a transverse magnetic field upon the second Townsend coefficient is discussed in greater detail than hitherto, and the possibility of differentiating by this means between the secondary processes operating is discussed.

I. Introduction

The mechanism of electrical breakdown of gases in uniform static electric fields is now well established in terms of the growth of current equations based on the first and second Townsend ionization coefficients, α and ω/α. The more complex situation in the presence of crossed electric and magnetic fields is not so well understood, however, and previous attempts (Wehrli 1922 ; Valle 1950 ; Somerville 1952 ; Haefer 1953) to explain the observed breakdown characteristics have not been entirely satisfactory. Each of these investigations has been concerned with the influence of a transverse magnetic field on one or both of the Townsend ionization coefficients.

Wehrli made calculations of the first Townsend coefficient in uniform crossed electric and magnetic fields, basing his theory on the assumption that all electron collisions with gas molecules are completely inelastic and that the free path l is constant for all electrons. In this case, an electron will describe a cycloidal path between collisions.

The distance l' travelled in the direction of the electric field E (V/cm) will then be

$$l'=l(1-eH^2l/8\times10^8Em), \quad \ldots\ldots\ldots\ldots \quad (\mathbf{1.01})$$

H being the magnetic intensity in oersteds, e and m the charge (e.m.u.) and mass (g) of an electron. When the magnetic field is absent, $l'=l$ and, since only the component of the free path in the direction of the field E affects the kinetic energy of the electron, Wehrli concluded that, in this sense, the effect of the magnetic field is equivalent to an increase in the pressure to a value p_e, where

$$p_e=p/(1-eH^2l/8\times10^8Em). \quad \ldots\ldots\ldots\ldots \quad (\mathbf{1.02})$$

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Valle inserted this " equivalent pressure " p_e into Townsend's approximate expression for α/p in static electric fields, namely,

$$\alpha/p = A \exp(-Bp/E), \quad \ldots\ldots\ldots\ldots \quad \textbf{(1.03)}$$

and so obtained an empirical relationship for α in a magnetic field. Neither of these authors considered the effect of the magnetic field on the secondary coefficient.

Valle's theory has been discussed in detail by Somerville (1952) and Haefer (1953) and they have shown that the neglect of the distribution of free paths about the mean is the cause of many of the major qualitative differences between the theoretical and observed values of sparking potentials in crossed electric and magnetic fields. By considering this distribution of free paths, Somerville obtained an improved form of the equivalent pressure concept, which is in better though not entirely satisfactory agreement with experiment.

As an alternative approach, Somerville (1952) and Haefer (1953) independently derived a new expression for α/p in crossed fields without resorting to the equivalent pressure concept. This derivation was based on the assumption of completely inelastic collisions and an ionization probability of unity for all those collisions for which the electron energy is greater than the ionization energy of the gas molecule. Their expression is

$$\alpha/p = A \sinh\{(a/2l)\sqrt{(1-4BL/Ea)}\}/\varphi(l/a)\sinh(a/2l), \quad \ldots \quad \textbf{(1.04)}$$

where a is the length of a complete cycloidal arch described by an electron starting from rest, i.e. $a = 8\times10^8 Em/eH^2$,
L is the mean free path at 1 mm Hg pressure ($L = pl$),
A, B are the empirical constants occurring in equation (**1.03**), and
$\varphi(x) = \coth(1/2x) - 2x$.

Furthermore, Somerville has shown that, provided $4BL/Ea < 1$, this expression for α/p leads to sparking potentials not greatly different from those deduced from his modification to Valle's theory, but that for $4BL/Ea > 1$ the theory breaks down completely, because the maximum energy gained over a cycloidal path by an electron starting from rest is then always less than the ionization potential of the gas, and, with the assumptions made, no ionization can then occur. In this case an adequate theory must take account of the possibility of an electron obtaining sufficient energy to ionize as the result of energy gained over several free paths.

Somerville (1952) and Haefer (1953) further extended their investigations to include the effect of a transverse magnetic field on Townsend's secondary coefficient ω/α, but limited their discussion to the case when positive ion action, either at the cathode or in the gas, is the only secondary process. In this case the problem is greatly simplified, since the magnetic field has little influence on the motion of positive ions in the gas and consequently their energies will not be changed appreciably except at very high magnetic field strengths. For this simplified case, the problem reduces to an investigation of the effect of the magnetic field on the secondary electrons produced at the cathode by the positive ions. The magnetic field causes these electrons to move in cycloidal paths and they may be recaptured by the cathode, thus effectively reducing ω/α.

All these previous theories have approached the problem through the study of individual electron trajectories in the gas. They have entailed some drastic simplifying assumptions, however, and, while qualitative agreement with experiment is reasonably good, quantitative agreement is far from satisfactory. This is hardly surprising when it is realized that, as yet, a satisfactory theory has not been derived on this basis even in the absence of a magnetic field. In view of this, a new approach is made to the problem in the present analysis and consideration given to the " bulk " properties of the electron avalanches, such as electron mean energy, drift velocity, and the distribution function for the electron energies.

In the absence of a magnetic field, this approach leads (Emeléus, Lunt, and Meek 1936) to the following expression for the first Townsend coefficient,

$$\alpha/p=KW^{-1}\int_0^\infty P(V).V^{\frac{1}{2}}.f(V).\mathrm{d}V, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{1.05})$$

where $K=\sqrt{(e/150m)}$ and is constant,
W is the electron drift velocity,
$P(V)$ is the ionization efficiency of electrons with energy V, at 1 mm Hg pressure, and
$f(V)$ is the distribution function of the electron energies.

In order to extend this approach to the case when a transverse magnetic field is present, it is necessary to determine the influence of the magnetic field on the quantities occurring on the right-hand side of equation (**1.05**). One may then obtain a new expression for α/p as a function of H/p and E/p.

II. The Influence of a Transverse Magnetic Field on the Electron Avalanche

In order to analyse the influence of the magnetic field on the properties of the electron avalanche, it is necessary to know the variation of the mean free path l with electron velocity u. The assumption usually made is that l is independent of u, and for some gases (notably air) this is approximately true. For hydrogen and helium, however, a more valid assumption is that l is proportional to u (von Engel 1955). Since the values of the Townsend coefficients in static electric fields are best known for hydrogen, experimental work in these laboratories has been carried out using this gas so that, in what follows, the assumption is made that l/u ($=T$, the mean free time) is constant for a given pressure. The error introduced by using this assumption for other gases will be discussed at a later stage.

When only the electric field E is present, any quantity Q under discussion will be denoted by $Q_{0,\,E/p}$; when both H and E are present, the quantity will be denoted by $Q_{H/p,\,E/p}$.

(a) The Electron Drift Velocities, $W_{0,E/p}$ *and* $W_{H/p,E/p}$

For the particular case when the mean free time is constant, the drift velocity $W_{0,E/p}$ is given by

$$W_{0,E/p}=EeT/m. \quad \dots\dots\dots\dots\dots\dots \quad (\mathbf{2.01})$$

This expression was first derived by Pidduck (1913) and has been obtained more recently by Davidson (1954) and Huxley (1957*a*), using different methods of derivation.

An expression for the corresponding drift velocity $W_{H/p,E/p}$ in the presence of a transverse magnetic field has been obtained by Huxley (1957*b*) assuming that $l=f(u)$, and, for the particular case when $l/u=T$, his result reduces to

$$W_{H/p,E/p}=\frac{E}{H}\,\frac{wT}{1+w^2T^2}, \quad \dots\dots\dots\dots\dots \quad (\mathbf{2.02})$$

where $w=He/m$.

This same result can be obtained by a rather different method, which is given in Appendix I, because this particular approach will be used later when discussing the influence of the magnetic field on the secondary coefficient.

From (**2.01**) and (**2.02**),

$$W_{0,E/p}/W_{H/p,E/p}=1+w^2T^2. \quad \dots\dots\dots\dots\dots \quad (\mathbf{2.03})$$

Also $wT=HeL/mpu$, so that for a given gas

$$w^2T^2=C(H/p)^2, \quad \dots\dots\dots\dots\dots \quad (\mathbf{2.04})$$

where C is constant $\{=(eL/mu)^2\}$.

Substituting for w^2T^2 in equation (**2.03**),

$$W_{0,E/p}/W_{H/p,E/p}=1+C(H/p)^2. \quad \dots\dots\dots\dots \quad (\mathbf{2.05})$$

(b) The Velocity Distribution

The precise form of the velocity distribution of electrons in crossed electric and magnetic fields is not known, except for the particular case when the collisions between electrons and gas molecules are elastic. Allis and Allen (1937) have shown that the distribution function can then be written as

$$f(u)=A_0u^2\exp\left\{-\frac{3m}{M}\left(2\int\varepsilon/\varepsilon_l^2.\mathrm{d}\varepsilon+[H^2/me^2]\int\mathrm{d}\varepsilon\right)\right\}, \quad \dots \quad (\mathbf{2.06})$$

where $\varepsilon=\frac{1}{2}mu^2$ and $\varepsilon_l=Eel$.

When the mean free time is independent of u, the integrals in equation (**2.06**) can be evaluated to give

$$f(u)=A_0u^2\exp\left\{-\frac{3m^3}{2M}\cdot\frac{(1+w^2T^2)}{E^2e^2T^2}\cdot u^2\right\}, \quad \dots\dots\dots \quad (\mathbf{2.07})$$

so that the distribution of electron velocities is Maxwellian. The distribution is Maxwellian whether the magnetic field is present or not, the effect of the magnetic field being the same as if T were replaced by $T/\sqrt{(1+w^2T^2)}$. Since

T depends only upon the *actual* gas pressure (at a given gas temperature) and $T=L/pu$, the effect of the magnetic field on the distribution function is equivalent to an increase in pressure by the factor $\sqrt{(1+w^2T^2)}$. Thus in this particular case, the magnetic field has no influence on the *form* of the distribution but only reduces the mean electron energy.

Whether this conclusion is valid when inelastic collisions occur remains to be determined. It is of interest in this connexion, however, to note the results of a recent redetermination of theoretical values of $(\alpha/p)_{0,E/p}$ in hydrogen (Blevin and Haydon 1957) showing that for this gas the distribution function may be assumed to be approximately Maxwellian. For hydrogen, then, the "elastic collision" theory cited above seems to fit the experimental results quite well even when inelastic collisions take place. Consequently, it will be assumed that the magnetic field does not affect the *form* of the distribution function, but has the effect of reducing the mean energy.

(c) *The Mean Electron Energy*

If the mean electron energy is $\overline{V}$ when both the magnetic and electric fields are present, then, for equilibrium, the average energy gain per free path must equal the average energy loss at collision, that is,

$$Ey=\lambda(\overline{V}).\overline{V}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.08)$$

where y is the average distance travelled in the direction of the field E between collisions, and $\lambda(\overline{V})$ is the average fractional energy loss at collision when the mean energy is $\overline{V}$. For a given gas $\lambda(\overline{V})$ depends only upon the distribution of energies, and consequently, using the assumption of the preceding section regarding the distribution function, will be independent of H.

Now

$$\begin{aligned} y&=T.W_{H/p,E/p} \\ &=\frac{L}{pu}W_{H/p,E/p}. \quad \ldots\ldots\ldots \quad (2.09) \end{aligned}$$

From (2.08) and (2.09)

$$\frac{E}{p}\cdot\frac{L}{u}\cdot W_{H/p,E/p}=\lambda(\overline{V}).\overline{V}. \quad \ldots\ldots\ldots\ldots \quad (2.10)$$

Since $\lambda(\overline{V})$ is independent of H, $\lambda(\overline{V}).\overline{V}$ may also be thought of as the average energy loss at collision in a different situation in which there is no magnetic field, and some different value of E/p prevails. Let this value be E/p'.

In this case, the equilibrium condition becomes

$$Ez=\lambda(\overline{V}).\overline{V}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.11)$$

where $z=T.W_{0,E/p'}$ or

$$z=(L/p'u).W_{0,E/p'}. \quad \ldots\ldots\ldots\ldots \quad (2.12)$$

From (2.11) and (2.12)

$$(EL/p'u).W_{0,E/p'}=\lambda(\overline{V}).\overline{V}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2.13)$$

Comparing (**2.13**) and (**2.10**) shows

$$p'/p = W_{0, E/p'}/W_{H/p, E/p} \quad \ldots\ldots\ldots\ldots \quad (\mathbf{2.14})$$

$$= \frac{W_{0, E/p'} \cdot W_{0, E/p}}{W_{0, E/p} \cdot W_{H/p, E/p}}.$$

Now from equation (**2.01**) $W_{0, E/p'}/W_{0, E/p} = p/p'$, so that

$$p'/p = \sqrt{(W_{0, E/p}/W_{H/p, E/p})}. \quad \ldots\ldots \quad (\mathbf{2.15})$$

Using (**2.05**)

$$p' = p\sqrt{\{1 + C(H/p)^2\}}. \quad \ldots\ldots\ldots \quad (\mathbf{2.16})$$

From the definition of p' it follows, then, that in crossed electric and magnetic fields the electrons behave energetically as they would if only the electric field were present, and the pressure were increased from p to p'.

Townsend and Gill (1938) have also derived an expression for the " equivalent pressure " by a consideration of the number of collisions made by an electron when advancing unit distance in the direction of the electric field. Since their result differs from equation (**2.16**), it is necessary to examine their work more closely.

When there is no magnetic field, the number (n_1) of collisions made by an electron moving unit distance in the direction of E is given by

$$n_1 = (W_{0, E/p} \cdot T)^{-1}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{2.17})$$

With the magnetic field present, the number of collisions becomes

$$n_2 = (W_{H/p, E/p} \cdot T)^{-1},$$

or, using (**2.05**)

$$n_2 = \{1 + C(H/p)^2\}/(W_{0, E/p} \cdot T). \quad \ldots\ldots\ldots\ldots \quad (\mathbf{2.18})$$

Townsend and Gill obtained this result and by comparing (**2.17**) and (**2.18**) concluded that the magnetic field has the same influence in this respect as would a decrease in T by the factor $1 + C(H/p)^2$, or, since $T = L/pu$, an increase in pressure by this factor. However, this makes no allowance for the change in the drift velocity which would take place if the pressure were increased.

Using equation (**2.01**), the result given in (**2.17**) becomes

$$n_1 = (u/L)p^2/E(e/m)(L/u). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{2.19})$$

Similarly (**2.18**) becomes

$$n_2 = (u/L)p^2\{1 + C(H/p)^2\}/E(e/m)(L/u). \quad \ldots\ldots \quad (\mathbf{2.20})$$

Comparing equations (**2.19**) and (**2.20**) now shows that the magnetic field is identical, in this respect, with an increase in pressure by a factor $\sqrt{\{1 + C(H/p)^2\}}$, in agreement with equation (**2.16**).

III. Calculation of $(\alpha/p)_{H/p,\,E/p}$

It is now possible to calculate $(\alpha/p)_{H/p,\,E/p}$ by way of equation (**1.05**), that is,

$$(\alpha/p)_{H/p,\,E/p}=K(W_{H/p,\,E/p})^{-1}\left\{\int_0^\infty P(V).V^{\frac{1}{2}}.f(V).\mathrm{d}V\right\}_{\bar{V}}, \quad \ldots \quad (3.01)$$

where the subscript $\bar{V}$ means that the integral is to be evaluated for this mean energy.

It also follows from equation (**1.05**) and the discussion of Section II (*c*) that

$$(\alpha/p)_{0,\,E/p'}=K(W_{0,\,E/p'})^{-1}\left\{\int_0^\infty P(V).V^{\frac{1}{2}}.f(V).\mathrm{d}V\right\}_{\bar{V}}, \quad \ldots \quad (3.02)$$

where p' is determined by equation (**2.11**).

From (**3.01**) and (**3.02**)

$$(\alpha/p)_{H/p,\,E/p}=\frac{W_{0,\,E/p'}}{W_{H/p,\,E/p}}\cdot(\alpha/p)_{0,\,E/p'}, \quad \ldots\ldots\ldots\ldots\ldots \quad (3.03)$$

or, using (**2.14**) and (**2.16**),

$$(\alpha/p)_{H/p,\,E/p}=\sqrt{\{1+C(H/p)^2\}}.(\alpha/p)_{0,\,E/p'}. \quad \ldots\ldots\ldots\ldots \quad (3.04)$$

For many gases and over a considerable range of the parameter E/p (von Engel 1955),

$$(\alpha/p)_{0,\,E/p}=A\exp(-Bp/E), \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3.05)$$

so that

$$(\alpha/p)_{0,\,E/p'}=A\exp[-B(p/E)\sqrt{\{1+C(H/p)^2\}}]. \quad \ldots\ldots \quad (3.06)$$

Equations (**3.04**) and (**3.06**) then yield

$$(\alpha/p)_{H/p,\,E/p}=A\sqrt{\{1+C(H/p)^2\}}\exp[-B(p/E)\sqrt{\{1+C(H/p)^2\}}]. \quad \ldots \quad (3.07)$$

Thus by comparing equations (**3.05**) and (**3.07**) it can be seen that the effect on the first Townsend coefficient of the addition of a transverse magnetic field is the same as an increase in pressure by the factor $\sqrt{\{1+C(H/p)^2\}}$.

Should the empirical relationship given in equation (**3.05**) be invalid for a particular gas, then equation (**3.04**) can be used, $(\alpha/p)_{0,\,E/p'}$ being evaluated from experimental results.

Also equations (**3.03**) and (**2.14**) show that

$$(\alpha/p)_{H/p,\,E/p}=(p'/p)(\alpha/p)_{0,\,E/p'}.$$

As these two equations depend only upon the assumption that the magnetic field does not alter the form of the distribution function, it would be expected that, if this assumption is valid, the " equivalent pressure " concept is justified for the general case when $L=f(u)$. Following the discussion of Section II (*b*), however, the assumption that the form of the distribution function is unchanged depends upon the condition L/u=constant, and it seems unlikely that these two assumptions can be divorced.

It should be emphasized that, in applying the results of the above analysis to pre-breakdown currents in uniform fields, a distinction must be made between the spatial and temporal variations, owing to the fact that the drift velocity, collision frequency, and consequently the electron transit time, are different in the following two systems:

$$\begin{array}{rll} \text{(i)} & \text{Magnetic field} & =H, \\ & \text{electric field} & =E, \\ & \text{pressure} & =p\,; \\ \text{and (ii)} & \text{magnetic field} & =0, \\ & \text{electric field} & =E, \\ & \text{pressure} & =p'. \end{array}$$

This means that although, as far as the primary ionization is concerned, the same electron multiplication takes place in a given distance, the time taken for this multiplication to be achieved is different in the two systems. This situation originates from the fact that when L/u is constant the average time spent between collisions depends only upon the actual gas pressure.

IV. Comparison of Theory and Experiment

The only measurements of $(\alpha/p)_{H/p,\,E/p}$ available for comparison with the theory are those made in hydrogen in these laboratories (Blevin 1956) for $50<E/p<150$ V cm^{-1} (mm Hg)$^{-1}$, and for H$<$700 oersteds. The hydrogen used in these experiments was admitted to the vacuum system by diffusion through a palladium thimble. The ionization chamber was not baked out, however, and, as a diffusion pump was not used, the lowest pressure obtainable in the system was about 10^{-3} mm Hg, so that the gas possibly contained small amounts of impurities.

The values of A, B used in (**3.07**) for the calculation of $(\alpha/p)_{H/p,\,E/p}$ were 5·6 and 141 respectively, corresponding to the measurements of $(\alpha/p)_{0,\,E/p}$ made in the same apparatus. These measurements are in good agreement with other recent determinations of $(\alpha/p)_{0,\,E/p}$ (Blevin, Haydon, and Somerville 1957).

The value of C to be used in equation (**3.07**) can be determined by several methods.

(i) Since $C=(eL/mu)^2$ and e, m are known, it is only necessary to find L/u. This may be calculated from collision cross-section data such as those determined by Ramsauer (1921) and Brode (1925).

(ii) Microwave measurements of the properties of electric discharges in hydrogen enable the collision frequency $\nu_c=u/L$ (at 1 mm Hg pressure) to be determined (Rose and Brown 1955; Udelson, Creedon, and French 1957), and hence C.

(iii) Equation (**2.01**) shows that $W_{0,\,E/p}=\sqrt{C}\,.\,E/p$, so that by taking a linear approximation to experimental values of the drift velocity, C and ν_c may be calculated. A summary of determinations of the electron collision frequency ν_c by these methods is given in Table 1.

That the values of ν_c obtained by these various methods are different is to be expected, because a collision between an electron and a molecule is defined differently for the different methods (Healey and Reed 1941). For the deter-

TABLE 1

COMPARISON OF COLLISION FREQUENCY MEASUREMENTS

Method	Author	$\nu_c \times 10^{-9}$ sec^{-1}
Collision cross-section data ..	Ramsauer	5·9
Collision cross-section data ..	Brode	3·6
Microwave measurements ..	Rose and Brown	4·85
Microwave measurements ..	Udelson, Creedon, and French	4·6
Drift velocities	(As in text)	3·6

TABLE 2

COMPARISON OF EXPERIMENTAL AND THEORETICAL VALUES OF $\alpha_{H/p,\,E/p}/\alpha_{0,\,E/p}$

$E/p=50$, $p=5\cdot17$ mm Hg

H/E	$\alpha_{H/p,\,E/p}/\alpha_{0,\,E/p}$ Expt.	$\alpha_{H/p,\,E/p}/\alpha_{0,\,E/p}$ Calc.
0·5	0·976	0·986
0·8	0·954	0·966
1·0	0·940	0·947
1·5	0·880	0·886
2·0	0·815	0·809
2·5		0·720
3·0		0·629

$E/p=70$, $p=5\cdot17$ mm Hg

H/E	$\alpha_{H/p,\,E/p}/\alpha_{0,\,E/p}$ Expt.	$\alpha_{H/p,\,E/p}/\alpha_{0,\,E/p}$ Calc.
0·5	0·988	0·985
1·0	0·940	0·942
1·5	0·855	0·873
2·0		0·789
3·0		0·597

$E/p=100$, $p=2\cdot52$ mm Hg

H/E	$\alpha_{H/p,\,E/p}/\alpha_{0,\,E/p}$ Expt.	$\alpha_{H/p,\,E/p}/\alpha_{0,\,E/p}$ Calc.
0·5	0·989	0·987
1·0	0·945	0·947
1·5	0·870	0·881
2·0	0·766	0·797
2·5	0·633	0·697
3·0		0·591
4·0		0·405
5·0		0·260

$E/p=150$, $p=2\cdot11$ mm Hg

H/E	$\alpha_{H/p,\,E/p}/\alpha_{0,\,E/p}$ Expt.	$\alpha_{H/p,\,E/p}/\alpha_{0,\,E/p}$ Calc.
0·5	1·02	1·001
1·0	0·985	0·989
1·5	0·895	0·937
2·0	0·754	0·855
3·0		0·637

mination of the effective cross section a collision is defined as an event in which an electron suffers an appreciable change either in direction of motion or in velocity, whereas for the drift velocity analysis, a collision is defined as an event in which, on the average, an electron loses all its momentum in any specified direction. Since the constant C has been introduced into the theory by a consideration of the electron drift velocities, it seems appropriate to select $\nu_c=3\cdot 6\times 10^9$, or $C=2\cdot 4\times 10^{-5}$.

Experimental and calculated values of $\alpha_{H/p,\,E/p}/\alpha_{0,\,E/p}$ are given in Table 2 for various values of E/p in the range $50<E/p<150$. Figure 1 shows the nature of the agreement between the present theory and the observed values, together with the theoretical values of Somerville and Haefer (cf. equation (**1.04**)), for $E/p=50$. It can be seen that even at large values of H/E, where the latter

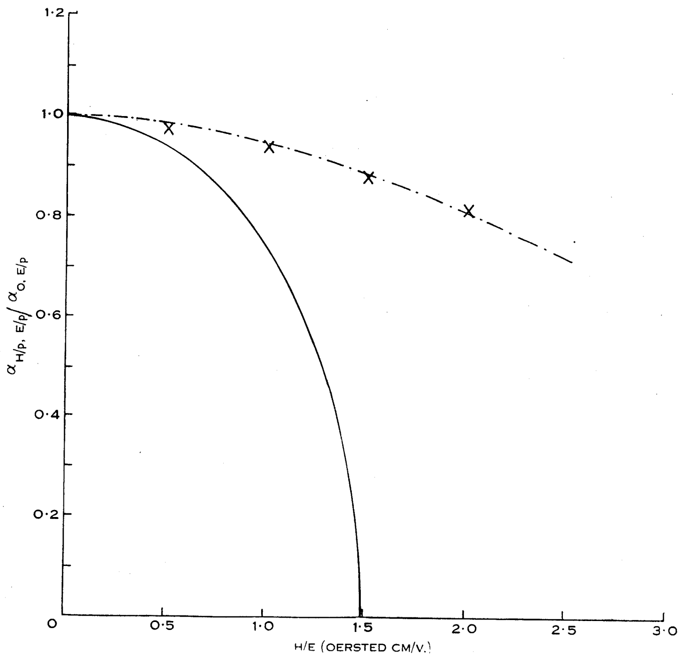

Fig. 1.—Theoretical and experimental values of $\alpha_{H/p,\,E/p}$ in hydrogen for $E/p=50$ V cm^{-1} (mm Hg)$^{-1}$. × Experimental values. —·— Present theory. —— Equation (**1.04**) (cf. Somerville 1952; Haefer 1953).

theory predicts zero values of $(\alpha/p)_{H/p,\,E/p}$, the present theory is in good agreement with observation. The calculated values are quite sensitive to the value chosen for ν_c because ν_c^2 appears in the calculation, so that the accurate measurement of $(\alpha/p)_{H/p,\,E/p}$ in pure hydrogen might well be used as the basis of the determination of ν_c in this gas.

V. Townsend's Secondary Coefficient

The secondary processes acting in an electric discharge can be denoted by a generalized coefficient ω/α (Llewellyn-Jones and Parker 1950). When only positive ion and photon action at the cathode are important,

$$\omega/\alpha=\gamma+\delta/\alpha, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{5.01})$$

where γ is the average number of electrons liberated from the cathode per incident positive ion and δ is the average number of electrons liberated from the cathode by the photons created in the gas when an electron moves 1 cm in the direction of the electric field. Equation (**5.01**) is valid, for instance, in low pressure discharges in hydrogen (Morgan 1956), for which gas it has already been shown in Section IV that the present theory for the first Townsend ionization coefficient is applicable.

Equation (**5.01**) does not take into account, however, the probability that electrons set free by either of these processes may be scattered back to the cathode after a collision with a gas molecule, and be captured there. If k is the fraction of secondary electrons which remain free in the gas, then

$$\omega/\alpha = k(\gamma+\delta/\alpha). \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5.02)$$

(*a*) *The Recapture Coefficient*, k

In the presence of both an electric and a transverse magnetic field there are *two* mechanisms by which electrons leaving the cathode may be returned there and recaptured.

(i) Electrons colliding elastically in the vicinity of the cathode can have sufficient energy to travel against the field E back to the cathode and be recaptured. Let the fraction of electrons which escape recapture by this mechanism be k_1. If only E is present, then, at those values of E/p where ionization is appreciable, k_1 remains nearly constant (Theobald 1953) with increasing E/p, so that little error is introduced by assuming that k_1 is independent of E/p, and, consequently, of the electron energy. Thus, although the presence of the magnetic field changes the mean electron energy, k_1 can be assumed independent of H for sufficiently high values of E/p.

(ii) There is a second loss mechanism which is not present in the absence of H, namely, loss of secondary electrons which do *not* suffer collision in the gas but return to the cathode under the action of the magnetic field. Let the fraction of electrons which escape recapture by this means be denoted by k_2. This process has been investigated by Somerville (1952), but slight modification of his theory is required when the mean free time rather than the mean free path is considered to be constant.

If t' is the time required for a secondary electron liberated from the cathode to return there, the number of electrons $N(t')$ travelling for this time without collision is (cf. Appendix I, equation (**A2**))

$$N(t') = N_0 \exp(-t'/T), \quad \ldots\ldots\ldots\ldots \quad (5.03)$$

where N_0 is the number of electrons leaving the cathode. Consequently the fraction of electrons remaining free in the gas is given by

$$1-N(t')/N_0 = 1-\exp(-t'/T),$$

and, if r is the probability that an electron will be reflected from the cathode, then from (**5.03**), $N_0 r \exp(-t'/T)$ electrons will leave the cathode again, and a fraction $r\{\exp(-t'/T)\}\{1-\exp(-t'/T)\}$ of these will collide and remain free in the gas. The value of k_2 is obtained by summing these fractions over an infinite number of reflections, giving

$$k_2=\{1-\exp(-t'/T)\}/\{1-r\exp(-t'/T)\}. \quad \text{(5.04)}$$

Now if it is assumed that the electrons leave the cathode with zero velocity, the distance x_t travelled in the direction of the electric field is (cf. Appendix I, equation (**A1**)),

$$x_t=\frac{1}{w}\frac{E}{H}(1-\cos wt),$$

but, by the definition of t', $x_t=0$ when $t=t'$, or

$$t'=2\pi/w. \quad \text{(5.05)}$$

From (**5.04**) and (**5.05**)

$$k_2=\{1-\exp(-2\pi/wT)\}/\{1-r\exp(-2\pi/wT)\}. \quad \text{(5.06)}$$

The coefficient k in equation (**5.02**) is then given by

$$k=k_1k_2. \quad \text{(5.07)}$$

(*b*) *The Variation of γ with Magnetic Field Strength*

For a given gas and cathode surface, γ will depend only upon the energy of the positive ions reaching the cathode. However, the magnetic field has little effect on the motion of positive ions in comparison to the effect on electrons, because of the much greater mass of the ions. Equation (**2.16**) shows (when C is evaluated for positive ions) that the "equivalent pressure" for the ions is very little different from the actual pressure, except for large values of H/p. With this restriction,

$$\gamma_{H/p,\,E/p}=\gamma_{0,\,E/p}. \quad \text{(5.08)}$$

The limiting value of H/p for which this is valid must be evaluated for each gas.

(*c*) *The Variation of δ/α with Magnetic Field Strength*

At low pressures when photon absorption in the gas is negligibly small, δ can be written in the form

$$\delta=\eta\theta g, \quad \text{(5.09)}$$

where θ is the average number of photons produced by an electron moving 1 cm in the direction of the electric field,
g is a geometrical factor determining the probability that a photon will reach the cathode, and
η is the probability that a photon reaching the cathode will liberate an electron.

By analogy with equation (**1.05**), the excitation coefficient θ is given by

$$(\theta/p)_{0,\,E/p}=K(W_{0,\,E/p})^{-1}\int_0^\infty P'(V).V^{\frac{1}{2}}.f(V).\mathrm{d}V, \quad \text{(5.10)}$$

where $P'(V)$ is the efficiency of excitation. Proceeding in the same manner as for ionizing processes, it follows that

$$(\theta/p)_{H/p,E/p}=(p'/p)(\theta/p)_{0,E/p'}. \qquad \textbf{(5.11)}$$

Now η will depend upon the energy of the incident photon and, when there are photons of different energies present, upon the relative abundance of photons in each energy group.

When excitations occur to different energy levels, $P'(V)$ in equation (**5.10**) can be replaced by $P'_1(V)+P'_2(V)+\ldots+P'_n(V)$, where $P'_n(V)$ is the excitation efficiency for the nth level. Thus, if $(\theta_j)_{0,E/p}$ is the number of photons produced which have energies characterized by the jth level of the gas molecule, then

$$(\theta_j/p)_{0,E/p}=K(W_{0,E/p})^{-1}\int_0^\infty P'_j(V).V^{\frac{1}{2}}.f(V).\mathrm{d}V,$$

and similarly,

$$(\theta_k/p)_{0,E/p}=K(W_{0,E/p})^{-1}\int_0^\infty P'_k(V).V^{\frac{1}{2}}.f(V).\mathrm{d}V,$$

so that

$$(\theta_j/\theta_k)_{0,E/p}=\int_0^\infty P'_j(V).V^{\frac{1}{2}}.f(V).\mathrm{d}V\bigg/\int_0^\infty P'_k(V).V^{\frac{1}{2}}.f(V).\mathrm{d}V. \qquad \textbf{(5.12)}$$

By a similar procedure, the ratio $(\theta_j/\theta_k)_{H/p,E/p}$ can be found, giving an equation similar to (**5.12**) but in which the integrals on the right-hand side must be evaluated for a different mean energy. It has already been shown that this mean energy corresponds to the case when there is only the electric field present, but the pressure is increased from p to p'. It follows, therefore, that,

$$(\theta_j/\theta_k)_{H/p,E/p}=(\theta_j/\theta_k)_{0,E/p'}. \qquad \textbf{(5.13)}$$

This result means physically that the relative abundance of photons in the energy groups is the same in the presence of both E and H, as in the case when only E is present, provided that the pressure is increased from p to p' in the latter case, that is,

$$\eta_{H/p,E/p}=\eta_{0,E/p'}. \qquad \textbf{(5.14)}$$

The value of g for a given electrode configuration depends only upon the manner in which the production of photons is distributed throughout the discharge space. In crossed fields, the number $\Delta n(x_1)$ of photons produced by N electrons moving a distance Δx in the direction of the electric field, having already travelled a distance x_1 from the cathode, is

$$\Delta n(x_1)=N\,.\,\theta_{H/p,E/p}\,.\,\Delta x.$$

If N_0 electrons originally leave the cathode, then

$$N=N_0\,.\,\exp\left\{\int_0^{x_1}\alpha_{H/p,E/p}\,.\,\mathrm{d}x\right\},$$

and

$$\Delta n(x_1)=N_0 \,.\, \theta_{H/p,\,E/p} \,.\, \exp\left\{\int_0^{x_1} \alpha_{H/p,\,E/p} \,.\, \mathrm{d}x\right\} . \,\Delta x.$$

Similarly the number of photons produced by electrons moving a distance Δx at a distance x_2 from the cathode is

$$\Delta n(x_2)=N_0 \,.\, \theta_{H/p,\,E/p} \,.\, \exp\left\{\int_0^{x_2} \alpha_{H/p,\,E/p} \,.\, \mathrm{d}x\right\} . \,\Delta x,$$

so that

$$\Delta n(x_1)/\Delta n(x_2)=\exp\left\{\int_{x_2}^{x_1} \alpha_{H/p,\,E/p}\mathrm{d}x\right\}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{(5.15)}$$

From (**2.16**), (**3.04**), and (**5.15**)

$$\Delta n(x_1)/\Delta n(x_2)=\exp\left\{\int_{x_2}^{x_1} \alpha_{0,\,E/p'}\mathrm{d}x\right\},$$

and it follows that the distribution of photon production in the gap when both E and H are present is the same as when only E is acting, but the pressure is increased from p to p', that is,

$$g_{H/p,\,E/p}=g_{0,\,E/p'}. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{(5.16)}$$

Combining equations (**2.16**), (**3.04**), (**5.09**), (**5.11**), (**5.14**), and (**5.16**),

$$(\delta/\alpha)_{H/p,\,E/p}=(\delta/\alpha)_{0,\,E/p'}, \qquad \ldots\ldots\ldots\ldots\ldots\ldots \quad \mathbf{(5.17)}$$

or, using (**5.02**), (**5.07**), (**5.08**), and (**5.17**),

$$(\omega/\alpha)_{0,\,E/p}=k_1\{\gamma_{0,\,E/p}+(\delta/\alpha)_{0,\,E/p}\}, \qquad \ldots\ldots\ldots\ldots \quad \mathbf{(5.18)}$$

and

$$(\omega/\alpha)_{H/p,\,E/p}=k_1k_2\{\gamma_{0,\,E/p}+(\delta/\alpha)_{0,\,E/p'}\}. \qquad \ldots\ldots \quad \mathbf{(5.19)}$$

VI. The Secondary Coefficient for Small H/p

Equation (**5.06**) shows that for small values of H/p the recapture coefficient k_2 is very close to unity, so that equations (**5.18**) and (**5.19**) can be written as

$$(\omega/\alpha)_{0,\,E/p}-(\omega/\alpha)_{H/p,\,E/p}=k_1\{(\delta/\alpha)_{0,\,E/p}-(\delta/\alpha)_{0,\,E/p'}\}. \qquad \ldots\ldots \quad \mathbf{(6.01)}$$

Furthermore, if the fraction of the secondary coefficient due to photons is known at any value of E/p, that is, the ratio $k_1(\delta/\omega)_{0,\,E/p}=f$ (say) is known, then equation (**6.01**) may be rewritten

$$(\omega/\alpha)_{H/p,\,E/p}-(1-f)(\omega/\alpha)_{0,\,E/p}=k_1(\delta/\alpha)_{0,\,E/p'}.$$

It follows then that if a value is assumed for f at a given E/p (obtained, for instance, from measurements of the formative time lag of breakdown (Morgan 1956)) it is possible, by measuring the total secondary coefficient ω/α with and without a transverse magnetic field present, to determine the photon contribution $k_1(\delta/\alpha)_{0,\,E/p'}$ at a value E/p'. By repeating this procedure the actual contribution due to photons can be determined at any lower value of E/p. In this way the fractional contributions obtained by measurements of the formative time lag of breakdown, in the absence of a magnetic field, may be checked.

VII. Conclusions

The above theoretical investigation has shown that the presence of a transverse magnetic field has the same effect on many of the " bulk properties " of an electron swarm in hydrogen as would an increase in the gas pressure. This equivalent increase in pressure has been determined and used to evaluate a theoretical expression for the first Townsend ionization coefficient in hydrogen which has been shown to be in good agreement with recent experimental determinations.

Table 3

Validity of Equivalent Pressure Concept

Property Associated with Electron Avalanche	Actual Situation: Transverse magnetic field $=H$, Uniform electric field $=E$, Gas pressure $=p$	Equivalent Situation: Transverse magnetic field $=0$, Uniform electric field $=E$, Gas pressure $=p'$
Distribution function	$f(V)$	$f(V)$
First Townsend coefficient	α	α
Collisions/cm of drift	n	n
Excitation coefficient	θ	θ
Photo-emission probability	η	η
Geometrical factor	g	g
Drift velocity	W	$(p'/p)W$
Collision frequency	ν_c	$(p'/p)\nu_c$

The detailed discussion of the secondary coefficient ω/α has also shown that, for small magnetic fields, the equivalent pressure concept is applicable to the influence of H on the photon contribution. It is not applicable to the influence of small magnetic fields on the liberation of electrons by positive ion bombardment of the cathode. As the magnetic field increases, the recapture of secondary electrons by the cathode becomes increasingly important, so that ω/α is diminished owing to this effect.

The extent to which the equivalent pressure concept developed in this analysis is valid is summarized in Table 3. It may be seen that the concept may be validly applied to all aspects of the spatial growth of currents which depend only on the first six quantities listed in the table. However, owing to the fact that the electron transit time is not identical in the two systems, an analysis of the *temporal* growth of pre-breakdown currents cannot be treated in terms of the particular value of the equivalent pressure derived in this analysis.

A further paper relating the present theory to breakdown characteristics in crossed fields is being prepared.

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Appendix I

The Drift Velocity of Electrons in a Transverse Magnetic Field

Consider an electron moving in a gas, at pressure p mm Hg, under the influence of a uniform electric field E in the $0x$ direction and a uniform magnetic field H in the $0z$ direction.

Putting $He/m=w$, the equation of motion for the electron is (see e.g. Healey and Reed 1941)

$$x_t=(1/w)[\{(E/H-u_y)\}\{1-\cos wt\}+u_x \sin wt], \quad \ldots\ldots \quad \text{(A1)}$$

where x_t is the distance travelled in the $0x$ direction in time t, and u_x, u_y are the initial velocities in the $0x$, $0y$ directions.

Now, if the mean free time T is independent of u, then the number $(\mathrm{d}N)$ of collisions made by N electrons in a time interval $\mathrm{d}t$, is

$$\mathrm{d}N=-N(\mathrm{d}t/T),$$

or

$$N=N_0 \exp(-t/T), \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad \text{(A2)}$$

where N_0 is the total number of electrons.

If a collision is defined as an event in which, on the average, an electron loses all of its momentum in any specified direction, then averaging for initial velocities in (**A1**) gives

$$\bar{x}_t = (E/Hw)(1 - \cos wt).$$

Averaging over free times, and using (**A2**),

$$\bar{x} = (E/H)wT^2/(1 + w^2T^2),$$

or

$$W_{H/p,E/p} = \bar{x}/T = (E/H)wT/(1 + w^2T^2).$$