THE THEORY OF STRONG KATABATIC WINDS

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Summary

In regions where strong katabatic winds occur, notably Commonwealth Bay, Antarctica, there is often a sharp boundary between the strong winds and the comparatively calm conditions at sea. The seaward movement of this boundary across a coastal station is accompanied by a sudden onset of strong winds and the landward movement by their sudden cessation. The boundary has all the characteristics of a pressure jump line accompanying a sudden change in depth of the cold air. Furthermore, it is to be expected from theoretical considerations that a pressure jump will occur near the coast. A simple theory is developed from which various characteristics of the flow can be predicted, including the intensity of the jump and the deviation of the wind from the lines of greatest slope caused by the Earth's rotation. These predictions agree moderately well with observations.

I. INTRODUCTION

According to the meteorological glossary katabatic winds blow down slopes that are cooled by radiation, the direction of flow being controlled almost entirely by orographic features. Winds of this type are best developed on the large ice caps of Greenland and Antarctica where the surface winds, apart from a slight deviation attributed to the Earth's rotation, almost always blow down slope, regardless of the direction. Near the foot of the ice slope these winds may be extremely violent and exhibit certain peculiarities which have excited comment from various authors, notably Mawson (1915), Madigan (1929), Mirrlees (1934), Prudhomme and Boujon (1952), and Boujon (1954). The peculiar features are perhaps best shown by the winds in the neighbourhood of Commonwealth Bay near the border of King George V Land and Adélie Land, Antarctica, though they occur at other places along the Antarctic coast and elsewhere. Only the principal features will be mentioned here; the reader is referred to the above-mentioned papers for a more detailed description.

The winds are extremely strong, persistent, and steady both in speed and direction. However, the striking feature is the occurrence of curious lulls during which the wind is light and variable. At the commencement of a lull the wind drops with great suddenness and almost simultaneously the pressure rises abruptly by 2–3 mb. Similarly, at the end of a lull the wind rises suddenly and the pressure falls abruptly. During the period of the lull strong winds may be both audible and visible higher up the slope and sometimes a strong airstream, rendered visible by drift snow, is seen overriding the calmer air beneath. A rotating roll cloud may also be present. These observations

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suggest the existence of a sharp boundary between the strong katabatic winds and the comparatively calm air over the sea. It will be shown subsequently that a sharp boundary is to be expected on theoretical grounds.

The main purpose here is the theoretical investigation of strong katabatic winds with particular reference to the retarding stage near the foot of the slope where the peculiar features have been observed. High up on the slope the air must pass through an accelerating stage. Probably before reaching the foot an approximate equilibrium is set up between the frictional resistance and the katabatic force, i.e. the wind approaching the coast is antitriptic. Near the coast the inertia of the wind plays a dominant part so that any satisfactory theoretical description of the retarding stage must take account of pressure forces, friction, and inertia. The problem will be simplified here by assuming the steady state, taking the upwind supply of cold air as given and supposing the potential temperature of the katabatic flow to be constant. The Earth's rotation can also have an appreciable effect on the flow but this will be neglected in the initial formulation of the theory, discussion being postponed to Section V.

II. Basic Equations

Pilot balloon observations indicate that the katabatic winds in the neighbourhood of Commonwealth Bay can be considered as a relatively thin layer of cold air flowing down the slope under the influence of gravity. For simplicity it will be supposed that the cold air has a definite depth, i.e. it is surmounted by a sudden inversion, and that the velocity and potential temperature of the cold air are independent of height. If there is no mixing across the inversion and cross-slope variations are neglected then the equation of continuity in the steady state takes the form

$$\frac{\mathrm{d}}{\mathrm{d}x}(hu)=0, \qquad \qquad (1)$$

where h is the height of the inversion, u is the velocity, and x is the horizontal distance measured downstream from an arbitrary fixed reference point. If in addition to the above assumptions vertical accelerations can be neglected, then the equation of motion can be written

$$\frac{\mathrm{d}}{\mathrm{d}x}(hu^2)=\frac{h\theta' g\alpha}{\theta}-\frac{h\theta' g}{\theta}\frac{\mathrm{d}h}{\mathrm{d}x}-\frac{h}{\rho}\frac{\mathrm{d}p}{\mathrm{d}x}-ku^2, \qquad (2)$$

where θ=potential temperature,

θ'=potential temperature deficit of the cold air,

α=angle of inclination of the ice slope (supposed small),

$\frac{\mathrm{d}p}{\mathrm{d}x}$=pressure gradient in the air above the inversion, i.e. the superimposed pressure gradient,

k=a dimensionless constant.

The last term on the right-hand side of equation (2) represents the friction force, supposed proportional to the square of the velocity, this being the generally accepted law for surface friction with moderate to strong winds. The other three terms arise from the horizontal pressure gradient on the assumption that the vertical distribution of pressure is hydrostatic. They will be referred to as the katabatic force, dependent on the slope, the gradient force, dependent on the gradient of h, and the force due to the superimposed pressure gradient. The sum of the first two terms vanishes when $\mathrm{d}h/\mathrm{d}x=\alpha$, i.e. when the inversion surface is horizontal. The direct effect of the superimposed pressure gradient in the neighbourhood of the foot of the slope is usually an order of magnitude smaller than the other terms and will henceforth be neglected. It may be taken into account by modifying α appropriately.

Equation (1) integrates immediately giving

$$hu=Q, \qquad (3)$$

where Q is a constant depending on the rate of supply of cold air from the ice plateau. Substitution in equation (2) gives an equation in h alone

$$\frac{\mathrm{d}h}{\mathrm{d}x}\left(h^3-\frac{\theta Q^2}{\theta' g}\right)=\alpha\left(h^3-\frac{k}{\alpha}\frac{\theta Q^2}{\theta' g}\right). \qquad (4)$$

This equation was first formulated and solved by Bresse in connexion with the flow of water in open channels. It has been considered in detail by hydraulic engineers who refer to its solutions as "backwater curves". Much of what follows in this section and Sections III and IV is based on standard hydraulic theory.

The critical depth h_c is defined by

$$h_c^3=\theta Q^2/\theta' g. \qquad (5)$$

If h equals h_c then the second factor on the left-hand side of equation (4) vanishes, therefore either $\mathrm{d}h/\mathrm{d}x$ is large and the equation is no longer valid since vertical accelerations cannot then be neglected, or the right-hand side vanishes, in which case α equals k and the slope is said to be critical. The flow will be termed tranquil or shooting according as h is greater than or less than h_c. In a region of flow throughout which equation (4) is valid the change from tranquil to shooting flow or vice versa can only occur where the slope is critical. In hydraulic applications it is found that equation (4) gives a sufficiently accurate description of the flow except in regions, covering only a very small range in x, where the surface changes abruptly from one level to another. These regions are known as jumps, and it will be shown in Section III that upstream of a stationary jump the flow is shooting and downstream it is tranquil. It can be seen therefore that a jump provides a means whereby the flow can change from shooting to tranquil at a point where the slope is not necessarily critical.

When the air has ceased to accelerate, $\mathrm{d}h/\mathrm{d}x$ is zero and the depth of the flow, called the normal depth h_n, is given by

$$h_n^3=k\theta Q^2/\alpha\theta' g. \qquad (6)$$

The uniform flow is shooting if $h_n<h_c$, i.e. if the slope is greater than the critical.

The preceding results can be expressed conveniently in terms of the Froude number, a dimensionless number defined as

$$F=\theta Q^2/\theta' g h^3. \qquad (7)$$

When the depth of the flow is critical F is unity, and the flow is shooting or tranquil according as F is greater or less than unity. The Froude number of the uniform flow is α/k and thus depends only on the steepness and roughness of the slope and not on the temperature or rate of supply of cold air. Various physical interpretations of the Froude number can be made. It can be regarded as the ratio of the inertia force, ρu^2, to the modified gravitational force, $\rho\theta' gh/\theta$, implying that the inertia of the flow dominates over gravity in shooting flow and vice versa in tranquil flow. It may also be regarded as the square of the ratio between the velocity, u, and the velocity of propagation of small amplitude gravity waves, $(\theta' gh/\theta)^{\frac{1}{2}}$, whence it follows that these waves cannot travel upstream in shooting flow. Finally it may be regarded as the reciprocal of a finite difference form of the Richardson number, i.e. with temperature gradient replaced by θ'/h and velocity gradient by u/h.

The stability of uniform flow has been investigated, among others, by Jeffreys (1925) with reference to flow of water in channels and by Defant (1933) with reference to air flow. It is found by simple perturbation theory that the flow is stable provided $F<4$. If $F>4$ then roll waves should develop. A description of this type of wave is given in Cornish (1934). Observations show that the roll waves do not develop except on slopes considerably steeper than indicated by the theory, partly because a very great length of slope is required for the waves to build up on slopes near the minimum.

Equation (**2**) expresses the momentum balance of the cold air ; the energy balance equation can be found by multiplying through by u and using equation (**3**) giving

$$-\frac{\mathrm{d}}{\mathrm{d}x}\left[\tfrac{1}{2}hu^3+\tfrac{1}{2}\frac{\theta' g}{\theta}h^2u+\frac{\theta' ghu}{\theta}(z+\tfrac{1}{2}h)\right]=ku^3, \qquad \text{(8)}$$

where $\mathrm{d}z/\mathrm{d}x=-\alpha$, whence z is the height of the surface above some arbitrary fixed reference level. The three terms in the square brackets on the left are respectively, the rate of transport of kinetic energy, the rate of working of the gradient force, and the rate of transport of potential energy. The total rate of decrease of energy is given by the rate of working of the friction force. In this case therefore the energy equation is implied by continuity and the equation of motion and no additional information can be obtained from it. It will be shown in Section III that this is not the case at a jump or sudden discontinuity in the flow.

III. Conditions at a Jump

Conditions at a jump may be investigated by applying conservation of mass and momentum to the flow in the neighbourhood of the jump. The change in depth of the cold air, though sudden, will occur over a finite distance which will be termed the jump width. It will be assumed that the changes in energy and momentum caused by the sudden change in depth outweigh the changes caused by surface friction and slope within the jump width. If the suffixes 1 and 2 are used to denote upstream and downstream values respectively, then, from continuity at a stationary jump aligned perpendicularly to the direction of flow,

$$h_1u_1=h_2u_2=Q, \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

and, by equating the force to the rate of change of momentum,

$$h_1u_1^2+\tfrac{1}{2}h_1^2\,\frac{\theta' g}{\theta}=h_2u_2^2+\tfrac{1}{2}h_2^2\,\frac{\theta' g}{\theta}=M. \quad \ldots\ldots\ldots\ldots (10)$$

Q is a known constant for the whole flow, both upstream and downstream of the jump, whereas the value of M is changed by slope and friction effects and is not known. The above relations therefore constitute three independent equations which enable any two of the four variables h_1, h_2, u_1, and u_2 to be eliminated.

Elimination of u_1 and u_2 leads to the symmetrical relation

$$h_c^3=h_1h_2(h_1+h_2)/2. \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Therefore if $h_1 \lesseqgtr h_c$ then $h_2 \gtreqless h_c$ and from these considerations a stationary jump can only occur where the flow changes from shooting to tranquil or vice versa, i.e. the flow must jump through the critical depth. For purposes of calculating the change in depth, given either upstream or downstream conditions, equation (**11**) may be solved for either h_1 or h_2 giving

$$\left.\begin{aligned} h_2&=\tfrac{1}{2}h_1[(1+8F_1)^{\frac{1}{2}}-1],\\ h_1&=\tfrac{1}{2}h_2[(1+8F_2)^{\frac{1}{2}}-1].\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (12)$$

For a given value of h_c, i.e. given values of Q and θ'/θ, then from equation (**12**) there corresponds to each depth h_1 a depth h_2 which will be denoted by a circumflex, thus $h_2=\hat{h}_1$ or $h_1=\hat{h}_2$.

The rate of change of mean flow energy per unit length of jump is given by

$$\Delta E=\rho\left[\tfrac{1}{2}h_2u_2^3-\tfrac{1}{2}h_1u_1^3+\frac{\theta' g}{\theta}(h_2^2u_2-h_1^2u_1)\right]=\frac{\rho Q^3(h_2-h_1)^3}{4h_1h_2h_c^3}. \quad \ldots (13)$$

Since energy cannot be created at a jump, ΔE must be negative, therefore $h_2>h_1$ and *a stationary jump can only occur as a transition from shooting to tranquil flow*. The mean flow energy which is lost at the jump either appears as turbulent kinetic energy or is radiated away by a stationary wave system situated downstream of the jump. For small hydraulic jumps the stationary wave system accounts for most of the energy whereas the energy appears as turbulence in intense jumps. These features of hydraulic jumps have been discussed by Benjamin and Lighthill (1954) and Binnie and Orkney (1955).

The equations for a stationary jump may readily be extended to apply to a moving jump, provided Q is replaced by Q_j the rate of flow through the jump. If the jump is moving downstream with a velocity c, then

$$Q_j=h_1(u_1-c)=h_2(u_2-c). \quad \ldots\ldots\ldots\ldots\ldots (14)$$

The upstream Froude number relative to the jump is F_{1j} say, where

$$F_{1j}=F_1(u_1-c/u_1)^2. \quad \text{(15)}$$

In order to calculate the change in height at a moving jump, F_1 must be replaced by F_{1j} in equation (12). It follows that, for given values of h_1 and F_1, a jump moving upstream is more intense than a stationary one and a jump moving downstream is less intense. Similar considerations apply if h_2 and F_2 are given. In general therefore, a jump moving upstream tends to be stronger and a jump moving downstream tends to be weaker than a stationary one.

The normal jump equations can also readily be extended to apply to a jump inclined to the direction of flow. Equations (9)–(13) are still valid provided u_1 and u_2 are taken as the components perpendicular to the jump line. The wind component parallel to the jump line is unchanged, whereas, at an intense jump, the component perpendicular to the jump is reduced almost to zero. In these conditions, even if the incoming flow is only inclined at a small angle to the normal, the outflowing air moves almost parallel to the jump line. The relationship between the inclinations to the normal, β_1 and β_2, before and after the jump can be deduced immediately from equation (12) by the substitution $h=Q/u=Q\tan\beta/v$, giving

$$\tan\beta_2=\tfrac{1}{2}\tan\beta_1\{(1+8F_1)^{\frac{1}{2}}-1\}. \quad \text{(16)}$$

IV. The General Solution

If α is not zero the general solution of equation (4) with α and k constant is

$$\alpha x/h_n=h/h_n+(1-F_n)\Phi(h/h_n), \quad \text{(17)}$$

where Φ is Bresse's Backwater Function given by

$$\Phi(s)=\frac{1}{6}\log\left\{\frac{s^2+s+1}{(s-1)^2}\right\}+\frac{1}{\sqrt{3}}\arctan\left\{\frac{s\sqrt{3}}{1-s}\right\}, \quad \text{(18)}$$

and F_n is the Froude number for uniform flow. This function is tabulated in Woodward and Posey (1941), Table 601. When α is zero the solution takes the particularly simple form

$$kx/h_c=h/h_c-\tfrac{1}{4}(h/h_c)^4. \quad \text{(19)}$$

The origin of x has been taken, for convenience, where h vanishes.

In a region throughout which equation (4) is valid, two boundary conditions are required to determine the solution completely, e.g. the value of Q and the depth h_0 at some known value of x, x_0 say. In practice for given Q this value of x is upstream in shooting flow and downstream in tranquil flow, because the behaviour of shooting flow is determined physically by upstream conditions and that of tranquil flow by downstream conditions. This is indicated when, for instance, an attempt is made to determine the depth some distance upstream in shooting flow where downstream conditions are given. It is found that very small differences in the given depth downstream can produce extremely large differences in the predicted depth upstream.

There are 12 different types of solution which can occur, only four of which will be considered here. These are:

(i) If $h_0>h_n>h_c$, then dh/dx is positive and tends towards α for large x, so that the inversion becomes horizontal downstream. For large negative x, h tends to h_n. The flow is tranquil.

(ii) If $h_0>h_c>h_n$, then dh/dx is positive and, as previously, the inversion surface becomes horizontal downstream. The depth decreases upstream until it approaches the critical depth where the assumed conditions no longer apply. The flow is also tranquil in this case.

(iii) If $h_0=h_n$, then dh/dx is zero and the flow is uniform. The flow may be either tranquil or shooting according to whether $h_n \gtrless h_c$.

(iv) If $h_c>h_0$ and $\alpha=0$, whence h_n is undefined, then once more dh/dx is positive and the depth increases downstream until it approaches the critical depth where the assumed conditions no longer apply. The flow is shooting.

Solutions (i), (ii), and (iv) are shown in Figure 1.

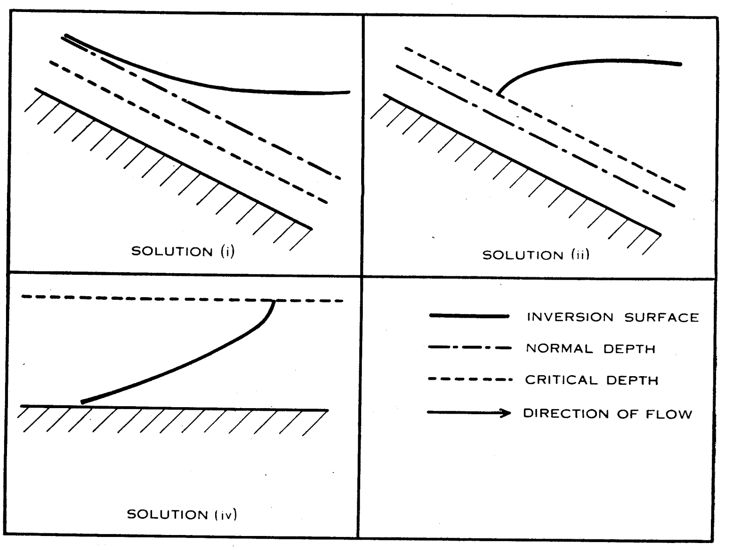

Fig. 1.—Backwater curves involving retardation

We are now in a position to determine the behaviour of the katabatic flow in the retardation stage near the coast. The rate of supply of cold air, Q, is taken as given and the flow is assumed to be uniform before it is retarded by coastal influences. The characteristic depth, H say, of the cold air over the sea is also taken as given. This depth cannot be predicted on the present theory since many of the factors which control the movement of the air over the sea have been neglected, e.g. the warming of the air by contact with the sea, friction from upper winds, and the effect of the Earth's rotation. These effects and others too will become important once the air has been retarded. It will also be assumed that $H>h_c$ since there appear, in general, to be no forces sufficient to maintain shooting flow at sea.

In the preceding paragraph three boundary conditions have been specified, the rate of flow Q, the upstream depth h_n, and the downstream depth H. If equation (4) were valid throughout the whole region one of these conditions would be unnecessary. However, the occurrence of a jump divides the flow into two separate regimes each of which, assuming Q known, requires one boundary condition. Furthermore, an additional condition is required to fix the position of the jump. This is provided by the jump equation (**12**) which relates the depths at the common boundary of the two regimes.

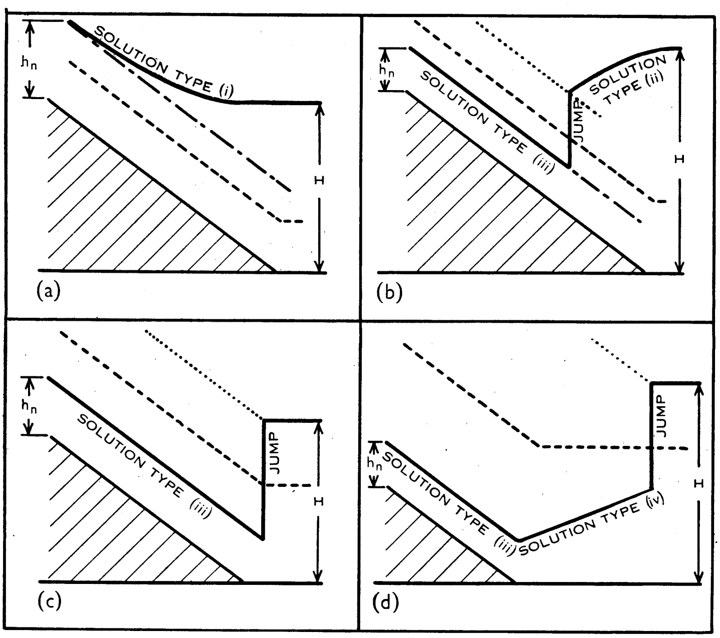

Fig. 2.—Flow types which can occur near the foot of a slope.
——— Inversion surface h. — — — — Normal depth.
- - - - - Critical depth h_c. ······· $\hat{h}$.
—→ Direction of flow.

The behaviour of the katabatic wind depends firstly on whether the uniform flow is shooting or tranquil. If the flow is tranquil, then no jump can occur and the air is retarded before reaching the coast, the depth of cold air being given by a solution of type (i). For given Q the solution is completely specified if H is known. The upstream condition $h=h_n$ is automatically satisfied for all solutions of type (i). A coastal station would in this case be in a region of light winds. In the neighbourhood of Commonwealth Bay this kind of flow does not usually occur because, as will be shown subsequently, h_n is normally less than h_c.

When the uniform flow is shooting a jump will occur near the coast. If the jump is situated inland then the flow between jump and coast will be type (ii) and the flow upstream of the jump will be normal, i.e. type (iii). A coastal station will once more be in a region of light winds since the air is retarded before reaching the coast. It is in this case that strong winds may be both audible and visible higher up the slope, as noted by various observers in both the Antarctic and Greenland. If the jump is situated out to sea then the flow between jump and coast will be of type (iv), the flow on the slope being uniform as before. It is in this case and this case only that strong winds occur at the coast. The flow types are shown in Figure 2.

When H equals $\hat{h}_n$ then the jump will be situated at the coast. Furthermore, it will be inland or at sea according as H is greater than or less than $\hat{h}_n$. The approximate position of the jump when it is near the coast can easily be calculated. Consider first the case $H > \hat{h}_n$. If H is the depth at the coast then it changes by an amount Δh at a distance inland from the coast given by

$$x = -\left(\frac{H^3 - h_c^3}{H^3 - h_n^3}\right)\frac{\Delta h}{\alpha}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

provided Δh is small. This is merely a restatement of equation (4) in a finite difference form. Immediately downstream of the jump h equals $\hat{h}_n$, so the distance of the jump from the coast is determined by putting Δh equal to $H - \hat{h}_n$ in equation (20). Similarly, when $H < \hat{h}_n$, the depth changes by Δh at a distance seaward from the coast given by

$$x = \left(\frac{h_c^3 - h_n^3}{h_c^3}\right)\frac{\Delta h}{k}. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (21)$$

Just upstream of the jump h equals $\hat{H}$, so the distance of the jump from the coast is obtained by putting Δh equal to $\hat{H} - h_n$ in equation (21). These relationships will be used to calculate jump positions in Section VI.

The various possible types of flow which can occur near the foot of a slope can be summarized as follows:

(*a*) $F_n < 1$ No jump occurs
(*b*) $F_n > 1$, $H > \hat{h}_n$ A jump occurs landward of the coast
(*c*) $F_n > 1$, $H = \hat{h}_n$ A jump occurs at the coast
(*d*) $F_n > 1$, $H < \hat{h}_n$ A jump occurs seaward of the coast.

The condition for type (*d*) is also the condition for strong winds to occur at the coast.

V. The Effect of the Earth's Rotation

The general equations governing the steady motion of a layer of cold air down a slope, taking account of the Earth's rotation, can be written

$$\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

$$\frac{\partial}{\partial x}(hu^2) + \frac{\partial}{\partial y}(huv) = \frac{h\theta' g}{\theta}\left(\alpha - \frac{\partial h}{\partial x}\right) - hvl - kVu, \quad \ldots\ldots \quad (23)$$

$$\frac{\partial}{\partial x}(huv) + \frac{\partial}{\partial y}(hv^2) = -\frac{h\theta' g}{\theta}\frac{\partial h}{\partial y} + hul - kVv, \quad \ldots\ldots \quad (24)$$

where y is the cross-slope coordinate, l is the Coriolis parameter, v is the cross-slope component of wind, and V is the wind speed, u being the down-slope velocity as before. No attempt will be made to find the general solution of this system of non-linear partial differential equations. Certain particular solutions can, however, be found very easily and two of these will be discussed briefly below.

To determine a solution appropriate to the Antarctic ice slope, cross-slope variations can be neglected, hu is then equal to a constant, as before, and the equations reduce to

$$Q\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{h\theta' g}{\theta}\left(\alpha-\frac{\mathrm{d}h}{\mathrm{d}x}\right)-hvl-kVu, \quad \ldots\ldots\ldots\ldots \quad (\mathbf{25})$$

$$Q\frac{\mathrm{d}v}{\mathrm{d}x}=Ql-kVv. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{26})$$

Multiplying (**25**) by u and (**26**) by v and adding gives

$$Q\frac{\mathrm{d}(\frac{1}{2}V^2)}{\mathrm{d}x}=+\frac{\theta' gQ}{\theta}\left(\alpha-\frac{\mathrm{d}h}{\mathrm{d}x}\right)-kV^3. \quad \ldots\ldots\ldots\ldots \quad (\mathbf{27})$$

When the flow is uniform it can be seen immediately from this equation that the normal speed V_n is given by

$$V_n^3=\alpha\theta' gQ/k\theta. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{28})$$

The speed of a uniform katabatic wind is therefore independent of the Earth's rotation. The air, however, no longer flows directly down the slope, consequently the down-slope component of the wind is reduced. It is this component which is relevant in determining the position of the jump when it is parallel to the coast; the position of the jump will therefore depend to a certain extent on rotational effects.

Having determined V_n it is simplest to express the other flow characteristics in terms of it. Thus

$$\sin\beta=\frac{V_n l}{g\alpha\theta'/\theta}, \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{29})$$

where β is the angle through which the flow is deflected by the Earth's rotation. This angle is determined by the ratio of the geostrophic acceleration $V_n l$ to the katabatic acceleration $g\alpha\theta'/\theta$, these quantities often being of comparable magnitude. The normal depth is given by

$$h_n=\frac{Q}{V_n\cos\beta}=\frac{Q}{V_n\{1-(V_n l\theta/g\alpha\theta')^2\}^{\frac{1}{2}}}. \quad \ldots\ldots\ldots \quad (\mathbf{30})$$

In this type of flow the deflection from the lines of greatest slope is such that the component of the katabatic acceleration perpendicular to the lines of flow balances the geostrophic acceleration. If $V_n l > g\alpha\theta'/\theta$ then no such balance can be set up, however great the deflection of the flow. In these circumstances uniform flow is impossible. This condition can be written

$$Q>(g\alpha\theta'/\theta)^2(k/l^3)=Q_0. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (\mathbf{31})$$

If $Q_0 \gg Q$, then rotation does not greatly affect the uniform flow. If Q_0 only just exceeds Q, then h_n is large and β is close to $\frac{1}{2}\pi$, indicating that the air tends to flow across the slope rather than down it.

The conditions under which the various types of flow occur, as listed at the end of Section IV, are altered by the introduction of rotation. As will be shown subsequently, these changes are only small in the neighbourhood of Commonwealth Bay but could be important under other conditions. The relevant Froude number for uniform flow is now $\alpha \cos^3 \beta/k$ instead of α/k, and the condition for a jump to occur parallel to the coast somewhere in the flow is $\alpha \cos^3 \beta/k > 1$. Therefore rotation always reduces the likelihood of a jump occurring. The condition for strong winds at the coast, i.e. the condition for flow type (d), is now

$$H < \tfrac{1}{2}h_n[(1+8\alpha \cos^3 \beta/k)^{\frac{1}{2}} - 1].$$

It can be seen that rotation decreases the quantity in the square brackets and, from equation (**30**), that it increases h_n. It is not clear from these simple considerations whether rotation will increase or decrease the tendency for strong winds at the coast. A more detailed analysis indicates that the tendency for strong winds is decreased for all values of β and α/k.

The preceding solution applies only to the free descent of air down a slope. If the air is constrained to flow in a certain direction by local topography, then the solution will no longer apply. Suppose, for instance, that the air is constrained to flow directly down slope, then, under uniform flow conditions, equations (**23**) and (**24**) reduce to

$$h_n \frac{\theta' \alpha g}{\theta} - k u_n^2 = 0, \qquad (\mathbf{32})$$

$$-\frac{\theta' g}{\theta} \frac{\mathrm{d}h}{\mathrm{d}y} + u_n l = 0. \qquad (\mathbf{33})$$

The solution is

$$h_n = \frac{l^2 \alpha \theta}{4k\theta' g} y^2,$$

$$u_n = \frac{l}{2} \frac{\alpha}{k} y,$$

giving

$$F_n = \frac{\alpha}{k}.$$

Therefore, although h_n and u_n vary across the slope, the Froude number remains constant. The geostrophic acceleration is here balanced by the change in depth across the slope, the air tending to pile up against whatever obstacle is constraining the flow.

VI. Comparison with Observation

In order to determine whether the katabatic wind near Commonwealth Bay is shooting or tranquil we may take as typical values: $h_n = 300$ m, $u_n = 30$ m sec^{-1}, and $\theta'/\theta = 1/60$, giving a Froude number of 18. There is consequently little doubt that the katabatic flow is shooting and a jump must therefore occur somewhere in the neighbourhood of the coast. The general explanation of the curious behaviour of the wind is now clear. The normal condition of strong steady winds experienced at the coast corresponds to flow of type (*d*). If the jump moves across the coast in response to an increase in H or a decrease in h_n, then lull conditions will prevail, corresponding to flow of type (*b*). Madigan's account of conditions during a lull, with strong winds both audible and visible higher up the slope, describes exactly what one would expect near a pressure jump (see Fig. 3).

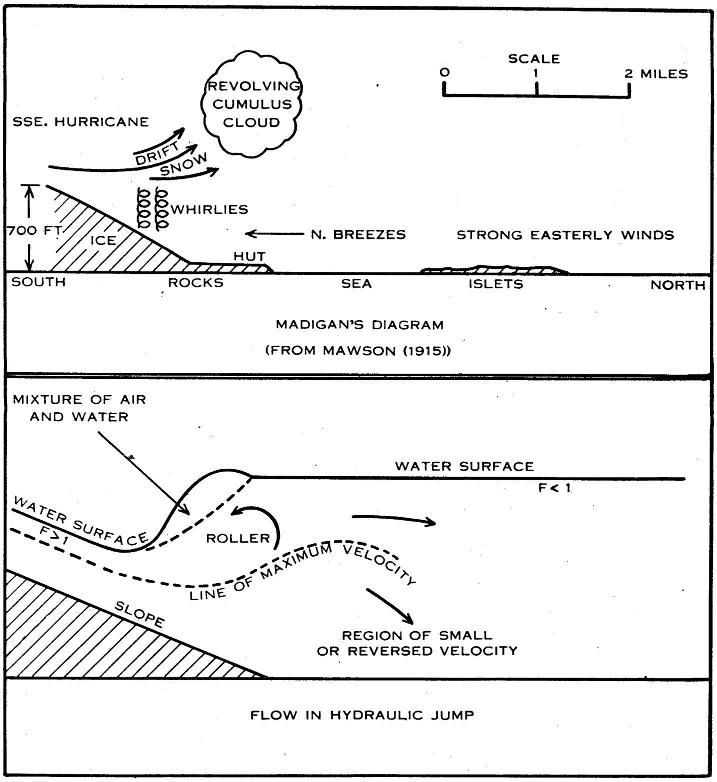

Fig. 3.—Comparison between Madigan's diagram and flow in a hydraulic jump.

As there is a considerable degree of uncertainty in the correct value for θ'/θ, we will take two values $0\cdot03$ and $0\cdot015$ corresponding to values for F_n of 10 and 20 respectively. In Commonwealth Bay α is about 10^{-1}. With these assumptions the following results are obtained :

F_n	k	h_c (m)	$\hat{h}_n$ (m)	Δp (mb)
10	10^{-2}	650	1200	$3\cdot2$
20	5×10^{-3}	800	1900	$2\cdot9$

Δp is the intensity of the pressure jump when it is situated at the coast. These values of k may be compared with a value of about 2×10^{-3} deduced from Liljequist's (1953) values for the roughness length on Antarctic ice. It is to be expected that the k value for the katabatic wind will be higher than the k value for surface friction alone since the wind is presumably retarded at its upper as well as its lower boundary. Near Commonwealth Bay the occurrence of strong winds at the coast indicates that the characteristic inversion height over the sea is generally less than $\hat{h}_n$. This is in agreement with a measured height of about 1000 m off the coast (Jalu 1950). The calculated pressure change is in good agreement with the observed change of about 3 mb. Furthermore, the pressure change at the commencement of a lull, when the jump is moving inland, is generally greater than the change at the end of a lull, when the jump is moving seawards (Boujon 1954). This confirms the prediction based on the theory of the moving jump (Section III).

If H exceeds $\hat{h}_n$ by 50 m then the jump will be situated about $\frac{1}{2}$ km inland (see equation (**20**)), whereas, if H is 50 m less than $\hat{h}_n$, the jump will be situated 2–4 km from the coast. Thus the seaward movement in response to a decrease in H is greater than the landward movement in response to a similar increase in H. This suggests that the jump will generally be most nearly stationary just inland of the coast and conditions there will be most suitable for the observational study of the jump.

The deviation of the wind from the lines of greatest slope caused by the Earth's rotation is 16° ($\theta'/\theta=0{\cdot}015$) and 8° ($\theta'/\theta=0{\cdot}03$) from equation (**29**). This compares with 10–15° observed at Cape Denison and about 5° observed at Port Martin. After passing through the jump the inclination will, according to equation (**16**), change from 16 to 60° in the first case and 8 to 30° in the second case. It is difficult to compare this with observation because the amount of turbulence generated at the jump is so great that typical turbulent velocities exceed the mean velocity downstream of the jump. It follows that the wind direction is extremely variable in that region, though further out to sea it generally settles down to an easterly. On the main Antarctic ice slope the principal effect of rotation is a slight deviation of the wind. Q_0 is about 10^5 m^2 sec^{-1} and probably exceeds any rate of flow that occurs in practice. However, on gentler slopes of equal or greater smoothness, with small inversion strengths, Q may quite often be greater than Q_0 and uniform katabatic winds will not then occur. For example, if α is 10^{-2} and the roughness and inversion strength remain the same as before, then Q_0 is about 10^3 m^2 sec^{-1}. This compares with the value for Q of about 10^4 m^2 sec^{-1} observed near Commonwealth Bay.

The predicted cross-slope velocity for uniform flow may be quite considerable, whereas the maximum cross-slope velocity that can develop when the air moves a distance x down slope is given by lx, i.e. $0{\cdot}15$ m sec^{-1} per km of movement. Thus in order to attain a velocity of 15 m sec^{-1} the air must move a distance of 100 km. In view of this it seems unlikely that uniform flow will be fully developed where the predicted cross-slope velocity is large. This idea can easily be translated into more precise terms. The required condition is

$$lx > v_n = V_n \sin\beta.$$

Substitution for V_n and $\sin\beta$ from equations (**28**) and (**29**) gives

$$x > (Q^2\theta/k^2\alpha g\theta')^{1/3}.$$

Whence, from equation (**5**),

$$x > h_c/(k^2\alpha)^{1/3} = \lambda \text{ say.}$$

An interesting feature of this inequality is the disappearance of the factor depending on the Earth's rotation. The values of λ corresponding to $\theta'/\theta = 0\cdot03$ and $\theta'/\theta = 0\cdot015$ are 30 and 60 km respectively. In the latter case the slope is probably not long enough for the development of uniform flow and the deviation near the foot of the slope will on these grounds be less than predicted on the assumption of uniform flow. In addition changes in the direction of the lines of greatest slope are liable to cause deviations which persist for considerable distances down slope. The observed deviation at any point will depend not only on the influence of the Earth's rotation but also on the irregularities of the slope upwind.

VII. References

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