THE STATISTICAL DESIGN OF AN EXPERIMENT TO TEST THE STIMULATION OF RAIN

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Summary

The problem of analysing statistically the results of a C.S.I.R.O. rainfall experiment over the catchment area of the Kiewa river is considered. It is shown that in order to obtain a significant result with any reasonable supposed increase in the rainfall within a few years it is necessary to use a control variable strongly correlated with the test variate. The powers of various tests using either streamflow or rainfall as test variates are calculated when the control variable is either streamflow in the Murray or rainfall in other areas. The general problem of designing such tests and the relationship between rainfall and streamflow are also considered.

I. Introduction

Since the latter part of 1954 rainfall stimulation using a silver iodide generator has been attempted by the C.S.I.R.O. Division of Radiophysics over the Kiewa area in Victoria. The generator is sited on Mount Stanley and the silver iodide is released whenever the wind is blowing into the 45° sector south to south-east of the generator. Decisive results in such an experiment cannot be expected for several years and the statistical analysis requires careful consideration. By making a preliminary analysis, however, we can determine how long such an experiment must be continued in order to have any specified probability of detecting a given increase in rainfall. Moreover, many experiments of this type have been carried out elsewhere without any idea of the statistical problems involved and have thus led to much wasted effort. It seems therefore worth while to discuss these problems at some length before the actual results of the experiment are obtained.

The statistical analysis involves taking some measured quantity X which it is hoped will be affected by the stimulation and deciding whether the observed difference between the mean values of X in the presence and absence of stimulation is larger than might be reasonably expected to have occurred by chance. Thus the analysis of the experiment is essentially a statistical one.

We must consider two characteristics of such a statistical test—its validity and its power. The test assumes a null hypothesis that there is no difference between the expected values of the observed variate in the presence and absence of stimulation. We then calculate a quantity, depending on the observed difference, whose statistical distribution is known under certain assumptions. The test will be a valid one if these assumptions are in fact true. In particular we have to make sure of the normality of the distribution of the test variate

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and of the absence of serial correlation and trend. If the latter are present a different form of analysis would be necessary.

We must also consider the power of the test, that is, the probability of obtaining a statistically significant result when a given increase in the expected rainfall occurs. Clearly, the greater the power of the test, the shorter the time necessary to make an adequate test of the method of stimulation. The power of the test will depend in part on the test criterion used and in part on the design of the experiment. The most effective way of increasing the power of the test is to use another variate (the " covariate ") which is unaffected by the seeding and which is strongly correlated with the test variate. Regression analysis then enables most of the variability of the latter to be eliminated and the power of the test is then greatly strengthened. In fact, detailed analysis shows that, without the use of such a covariate, experiments to test rain stimulation would have to be impracticably long.

We also have to choose the test variate. In the present case there are two such variates which it would be natural to choose—the river flow of the Kiewa River, and the rainfall in the test area. We shall consider the analyses of these separately and then consider what can be said about their relative merits. In both cases we shall take annual values. It might, at first sight, be thought better to use monthly or even weekly values. It is easy to show, however, that so long as a sufficient number of observations are used to obtain a good estimate of the standard deviations and correlations involved, there is no increase in power by using shorter intervals. Moreover, shorter intervals produce many more troubles in the shape of seasonal variation, non-normality, and serial correlation.

It has, however, been pointed out to me by a meteorologist friend that only part of the annual rainfall is received from winds blowing in the directions considered. He estimates, roughly, that Bright and Omeo, for example, receive about 40 and 30 per cent. respectively of their annual rainfall from winds between north and north-west. Thus, assuming rainfall is not correlated with wind direction, to produce a 10 per cent. increase in annual rainfall, the seeding operation would have to increase rain by 25–33 per cent. on the occasions on which it is used. This reduces the power of the test relative to a given increase in rainfall. In what follows, in speaking of a given percentage increase we shall mean that percentage increase in the target area and not the maximum possible increase which might be obtained if the target area had been seeded from all directions. These considerations suggest that a more powerful test might be obtained by confining the data to cases when the wind was in the right direction. This might be done but would require a considerably more complicated analysis and would in any case not be applicable to the case where stream flow is the test variate. In the present paper we therefore confine ourselves to annual data.

II. Analysis with River Flow as Test Variate

We define x_1 as the annual river flow at Kiewa. This is known from 1886 onwards but we shall only use the values from 1891 since we are going to correlate it with x_2, the annual flow of the Murray at Jingellic, which is only known from

1891 onwards. Here and in what follows we consider the period of 58 years from 1891 to 1948. In the final analysis of the experiment we will have further years added to these and we give in this paper all the sums, sums of squares, and sums of products of the variates used in order to reduce the work of future analysis. For the 58 years used we have, in thousands of acre feet,

$$\Sigma x_1 = 30125, \qquad \bar{x}_1 = 519\cdot397,$$
$$\Sigma x_1^2 = 19672017, \qquad \text{S.D. } (x_1) = 265\cdot74.$$
$$\text{Coefficient of variation of } x_1 = 0\cdot51.$$

Both the series x_1 and x_2 have somewhat skew distributions and they were therefore transformed by taking logarithms to the base 10 to four decimal places. We write $x_3 = \log_{10} x_1$ and $x_4 = \log_{10} x_2$.

Frequency distributions of x_3 and x_4 are shown in Tables 1 and 2.

TABLE 1

FREQUENCY DISTRIBUTION OF x_3

Values ..	2·2–	2·4–	2·6–	2·8–	3·0–	3·2–3·4	Total
Frequency ..	6	14	24	12	1	1	58

TABLE 2

FREQUENCY DISTRIBUTION OF x_4

Values ..	0·6–	0·8–	1·0–	1·2–	1·4–	1·6–1·8	Total
Frequency ..	1	4	19	26	7	1	58

A normal distribution was fitted to x_3 and the observed fit was good. This gives some confidence that residuals from the regression of x_3 on x_4 are normally distributed. Taking the values from 1891 to 1948 we get:

$$\Sigma x_3 = 154\cdot6147, \qquad \Sigma x_4 = 71\cdot2300,$$
$$\Sigma x_3^2 = 414\cdot71706619, \qquad \Sigma x_4^2 = 89\cdot46846034,$$
$$\Sigma x_3 x_4 = 191\cdot92945407, \qquad r_{34} = 0\cdot9084.$$

The high value of the correlation between x_3 and x_4 shows that x_4 is a useful and satisfactory covariate. Let $z_1, \ldots, z_n$ be the values of x_3 for the n years during which the area is seeded and $w_1, \ldots, w_n$ the corresponding values of x_4. We then test whether the mean of $z_1, \ldots, z_n$ is significantly larger than that of x_3 when the effects of x_4 and w are removed (the above values extend only to 1948 and it would be as well to add the values for 1949–1953). The analyses of the experiment thus takes the form of a standard analysis of variance and covariance.

In the above data we already have 58 years of records so that r_{34} and the variances can be fairly well estimated. We can therefore use a rough calculation to determine, for any given number of years of seeding and any given supposed increase in streamflow, the probability of obtaining a significant result at a prescribed significance level, i.e. we can determine the power function of the test.

The standard deviation of x_3 is $0 \cdot 2114$ so that the standard deviation of x_3 corrected for x_4 is approximately $0 \cdot 2114(1-r_{34}^2)^{\frac{1}{2}}=0 \cdot 08838$. The standard deviation of the difference between the mean of the n years of seeding and the 63 years (1891–1953) which will be used in the final test is about

$$0 \cdot 08838(n^{-1}+63^{-1})^{\frac{1}{2}}.$$

We are here considering a one-sided test so that we judge the observed difference of means to be significant at the 5 per cent. level if it exceeds $1 \cdot 6449$ times its standard deviation.

TABLE 3

PROBABILITIES OF SIGNIFICANT RESULT (5% POINT)

Increase (%)	Years		
	2	3	4
10	0·16	0·20	0·23
20	0·35	0·45	0·54
30	0·56	0·70	0·80
40	0·74	0·88	0·94
50	0·87	0·96	0·99
0	0·05	0·05	0·05

Increases of 10, 20, . . ., 50 per cent. in the mean streamflow will result in increases in the expected value of x_3 of $0 \cdot 04139$, $0 \cdot 07918$, $0 \cdot 11394$, $0 \cdot 14613$, and $0 \cdot 17609$ respectively. Using these values we calculate the probabilities given in Table 3 of judging the increase to be significant at the 5 per cent. point when the experiment lasts 2, 3, or 4 years. The method of calculation may be illustrated for the case of a 30 per cent. increase and a period of 3 years. The standard deviation used in the test is $0 \cdot 08838(3^{-1}+63^{-1})^{\frac{1}{2}}=0 \cdot 0522$ and multiplying this by $1 \cdot 645$ we get $0 \cdot 0859$. The mean of 3 years has a standard deviation (corrected for the regression) of $0 \cdot 0510$ and an expected value of $0 \cdot 1139$ so that its chance of exceeding $0 \cdot 0859$ is given by tables of the normal distribution and is $0 \cdot 70$ approximately. These results are only approximate as the above method of calculation is not that which would be used in the final analysis. Longer periods would be required to obtain the same probabilities of obtaining results significant at the 1 per cent. level.

These results are perhaps not very encouraging when it is realized that we are here working in the rather fortunate circumstances of having a covariate whose correlation with the test variate is about $0 \cdot 9$, but they emphasize the fact that without a control variate experiments on data of this degree of variability would have to be very long indeed.

One possible way of improving the power is to use more than one control variate. An attempt to do this was made by taking a number of rainfall stations north and north-west of the target area. These were Albury, Rutherglen, Chiltern, Wangaratta, Benalla, and Tallangatta. All of these have uninterrupted records dating back to 1891 or earlier. As a control variate their sum, x_5, was taken. For the 58 years 1891–1948 we have:

$$\Sigma x_5 = 9065\cdot 00, \qquad \Sigma x_5^2 = 1505538\cdot 0460,$$
$$\Sigma x_3 x_5 = 24548\cdot 246409, \qquad \Sigma x_4 x_5 = 11499\cdot 274353.$$

Hence

$$r_{35} = 0\cdot 8053,$$
$$r_{45} = 0\cdot 8720.$$

Hence

$$r_{35\cdot 4} = 0\cdot 0639,$$

which is not significant at the 10 per cent. point, and so the use of x_5 as a second covariate is not of any value.

We must also consider whether there is any trend in the series such as would arise from slow climate changes. That such climatic changes occur is well known (see for example Cornish (1954)). In the present problem such a trend, if it exists, may have little effect on the validity of the test because it would be likely to affect both the Kiewa and Jingellic flows more or less equally and we are only concerned with the residuals from the regression. Taking time as a new variate x_6 we get $r_{36} = 0\cdot 0704$ and $r_{46} = 0\cdot 0206$, neither of which is significant at the 10 per cent. point.

The above tests on correlation coefficients also assume the absence of serial correlation from year to year. The latter, if it exists, might seriously upset the test of significance of the experiment. The use of stream flow as a test variate, in contrast to rainfall, gives ri[illegible]o a suspicion that such a correlation may exist because of the lagged run-off wnich results from single storms and also from the fact that snow may lie for some time. To ensure the validity of the final test we need to ensure that the residuals from the regression on the covariate are uncorrelated and we must therefore test these residuals for serial correlation. A one-sided test will be appropriate since such correlation as may exist is not likely to be negative. Notice that it is the serial correlation in the residuals rather than in the original series which is important.

An exact and asymptotically most powerful test for serial correlation in residuals from a regression is now available (Hannan 1955). However, a simpler approximate method which could be applied here is given by Durbin and Watson (1950, 1951). Such a test should be applied before the final analysis is made.

The tests for trend and serial correlation will answer, in part, any criticism that the records are not homogeneous. Such inhomogeneity might arise if (1) the earlier records were not obtained with the same standard of accuracy or by the same method as the later records or if (2) there are local inhomogeneities in the series due to some specific meteorological factor which operates over a number of years and then disappears. The absence of trend in the series seems to indicate that (1) is not present to any significant effect and (2) also does not seem to appear and seems unlikely on other grounds. If there is a genuine effect of type (2) it should appear in the test of serial correlation. These statements apply also to the rainfall series.

It seems best therefore to go ahead and analyse the final experiment on the assumption that these disturbing factors are unimportant. If, however, convincing evidence can be produced to show that the series are sufficiently inhomogeneous to make the final analysis doubtful, another method of analysis can be used which, although slightly less powerful, will avoid these difficulties. This is done by truncating the series at 1945 (say) and testing whether the results for the test period (1955 onwards) are significantly larger than the means for 1946–53 (say) when the variances and covariances are estimated from 1891–1945 only. The above effects, if they exist, should then only have the effect of inflating the variances and the test will still be valid.

III. The Analysis with Rainfall as a Test Variate

The only rainfall stations within the target area and not too far from Mount Stanley which have long and uninterrupted records are Bright (from 1881) and Omeo (from 1880). At first it was thought that the sum of these would make a good test variate after a logarithmic transformation and using only the years 1891–1948 since this was the period for which the other observations were available. This was done and the resulting variate was found to have an observed distribution which is well fitted by a normal distribution. As a control variate $x_9 = \log_{10} x_5$ was taken (x_5 being the sum of the rainfalls at the six stations previously mentioned). The correlation with x_9 was found to be $0 \cdot 9525$. This is remarkably high. Such a high correlation being somewhat unexpected it was decided to see what sort of correlation was to be expected between rainfall stations in such an area. As an experiment, therefore, the six control stations were split into two groups of three, Albury, Rutherglen, and Chiltern forming one group and Wangaratta, Benalla, and Tallangatta the other. The correlation between the sums of rainfall in these two groups, without using a transformation, was $0 \cdot 9524$ for the 58 years. This suggests that such high correlations are not unusual.

A correlation $0 \cdot 9524$ between the test and control variates is very satisfactory and suggests that a powerful test would result. However, both Dr. C. H. B. Priestley and Mr. E. B. Pender have pointed out to me that Omeo is in a rain shadow on the south-eastern side of the mountain range and is therefore, presumably, less sensitive to any increase in rainfall which might result from seeding from Mount Stanley. It was decided, therefore, to use the figures for Bright alone and define the annual rainfall there as x_7. In any final analysis it would, of course, be interesting to see what happens to the Omeo rainfall as well.

To avoid skewness in the distribution we use a logarithmic transformation and define $x_8 = \log_{10} x_7$. For this variate we have the frequency distribution given in Table 4. As a control variate we take $x_9 = \log_{10} x_5$, which has a frequency distribution shown in Table 5.

TABLE 4

FREQUENCY DISTRIBUTION OF x_8 (1891–1948)

Values ..	1·35–	1·40–	1·45–	1·50–	1·55–	1·60–	1·65–	1·70–	1·75–	1·80–	1·85–1·90
Frequency ..	2	3	3	8	10	9	12	9	1	0	1

TABLE 5

FREQUENCY DISTRIBUTION OF x_9 (1891–1948)

Values ..	1·90–	1·95–	2·00–	2·05–	2·10–	2·15–	2·20–	2·25–	2·30–	2·35–	2·40–2·45
Frequency ..	2	3	1	6	9	12	12	4	5	2	2

These distributions are well fitted by normal distributions. We have:

$$\Sigma x_8 = 93\cdot2346, \qquad \Sigma x_9 = 126\cdot4705,$$
$$\Sigma x_8^2 = 150\cdot485718, \qquad \Sigma x_9^2 = 276\cdot45548583,$$
$$\Sigma x_8 x_9 = 203\cdot90216,$$
$$r_{89} = 0\cdot9307, \qquad (1 - r_{89}^2) = 0\cdot1338.$$

The standard deviation of x_8 is $0\cdot10360$ and so the standard deviation of x_8 corrected for x_9 is about $0\cdot03790$. The standard deviation of the mean of n years is then $0\cdot03790 n^{-\frac{1}{2}}$ and so the standard deviation of the difference of this mean and the mean of the 63 years (1891–1953) to be used in the final test will be $0\cdot03790(n^{-1}+63^{-1})^{\frac{1}{2}}$. As we are using a one-sided test, we take the observed

TABLE 6

PROBABILITIES OF OBTAINING SIGNIFICANT RESULTS FOR GIVEN PERCENTAGE INCREASE IN RAINFALL

Increase (%)	Years		
	1	2	3
10	0·29	0·45	0·58
20	0·67	0·90	0·97
30	0·91	0·995	1·00

mean difference to be significant at the 5 per cent. level if it exceeds $1\cdot645$ times its standard deviation. For increases of 10, 20, and 30 per cent. in the expected values of x_7 we get the following approximate probabilities of obtaining a result significant at the 5 per cent. level (Table 6).

These results are approximate and the correct form of the final test is that of an analysis of variance and covariance together with a t-test.

IV. The Relationship between Rainfall and Run-off

The above calculations show that a given percentage increase in rainfall has a much higher probability of being detected than the same percentage increase in run-off. This is, of course, partly due to the fact that the correlation with the control variate is higher in the first case (0·9307) than in the second (0·9084). However, it is also due to the fact that the variation of the streamflow is larger compared with its mean than that of rainfall. This is clear from the coefficients of variation which are 0·51 for x_1, the streamflow at Kiewa, and 0·24 for x_7, the Bright rainfall. Thus a 20 per cent. increase in streamflow is smaller compared with the standard deviation of the latter than a similar increase in rainfall.

Since streamflow is the result of rainfall one would expect at first sight that the coefficients of variation of the two should be about the same. Furthermore, if one takes rainfall at only one or two stations in the target area one would expect the " error " resulting from taking this as a measure of the total precipitation on the area to have the effect of making the coefficient of variation of rainfall appear larger than that of streamflow. In the above results just the reverse is the case. This is due to the fact that run-off is an " excess " remaining after a certain amount of the rainfall has been lost by evaporation, transpiration, and seepage (see for example the discussion in Johnstone and Cross (1949, pp. 103–5)).

If this is the correct explanation it would be quite misleading to compare the powers of the two test variates by comparing the probabilities of obtaining a significant answer for the same annual percentage increase of each because a 20 per cent. increase (say) in rainfall will probably cause a much larger percentage increase in run-off. To make a useful comparison of the probabilities of detecting a 20 per cent. increase in rainfall when using rainfall or run-off would require a much more elaborate calculation. It may well turn out that run-off provides as powerful or nearly as powerful a test as rainfall and so both methods should be used in the final analysis.

The above-mentioned relationship between rainfall and run-off also suggests that the economic effects of a 20 per cent. (say) increase in rainfall may be much larger than those ascribable to a 20 per cent. increase in run-off.

V. Conclusions

(1) Both rainfall and streamflow will be useful test variates when associated with suitable covariates.

(2) The experiment should be continued for several years and will then provide a decisive test of the practicability of rain stimulation by these methods.

(3) Provided previous records exist for a long enough time in the past for the estimation of variances and correlation, annual values are to be preferred to monthly or weekly values.

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VII. References

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