THE EFFECT OF MULTIPLE SCATTERING ON ELECTRON ENERGY LOSS DISTRIBUTIONS

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Summary

Calculations have been performed to obtain the theoretical energy loss distributions of 1 MeV electrons traversing foils thick enough for multiple scattering to be important. A criterion has been developed and applied to previous experiments to determine the foil thicknesses for which multiple scattering has an appreciable effect on the energy loss distribution.

I. INTRODUCTION

The problem of the distribution of energy losses in the passage of high energy electrons through foils has recently been studied by several workers (Paul and Reich 1950 ; Chen and Warshaw 1951 ; Kalil and Birkhoff 1953 ; West 1953 ; Hungerford and Birkhoff 1954). Under various conditions of foil thickness and electron energy, these workers have found values for the most probable energy loss which agree reasonably well with the theories of Landau (1944) and of Blunck and Leisegang (1950). The Blunck-Leisegang distribution is, however, appreciably wider than that of Landau for thin foils, but the correction to the Landau distribution becomes less significant as the foil thickness is increased whereas the difference between the experimental and theoretical widths increases with thickness. Most explanations of this difference have been qualitative statements that multiple scattering would increase the theoretical widths. As a step towards clearing the confusion that exists, it has been thought desirable to determine the extent to which multiple scattering affects the energy loss distribution of electrons and to apply these predictions to previous experiments. In conjunction with this, an experiment has been performed under conditions similar to those used in the calculations. A comparison will be made with this experiment in an accompanying paper (McDonell, Hanson, and Wilson 1955).

II. THE MONTE CARLO METHOD

(*a*) *General Procedure*

The C.S.I.R.O. Mark I Digital Computer was used to determine the paths of particles obeying one of two multiple scattering theories : that of Rossi and Greisen (1941) adapted to the three-dimensional case, and that of Molière (1948). The Molière theory should be more accurate, so most of the calculations were based on this, but two cases were worked out using the Rossi-Greisen theory

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for comparison (see Table 2 and Figs. 1 and 2). These multiple scattering theories are applied to short sections of fixed length l along the path of the particle. In each section the particle is considered to go straight until it reaches the end of the section when the direction of travel and the energy are changed for the next section. Each particle is incident normally on the scattering foil with an energy of 1 MeV and is followed through the foil until the energy falls below 0·4 MeV or until the particle escapes from the foil. Particles whose energy falls below 0·4 MeV are considered lost as the scattering at that energy is becoming excessive. Also classified as lost are those particles leaving the source side of the foil.

In common with other methods, the Monte Carlo method requires an accurate knowledge of the whole angular distribution for scattering. The small angle approximation used in multiple scattering theory means that the probability of large deflections, though small, is not necessarily accurate. A large deflection usually means a greatly increased path length and thus a large energy loss. The Monte Carlo distribution should therefore be accurate for small and medium energy losses but not necessarily accurate in predicting the probability of large energy losses or the stopping of electrons in the foil.

(*b*) *Choice of Energy Loss Distribution*

The energy loss distribution chosen was that of Landau because it is more convenient mathematically. The angular distribution of scattered electrons depends on their energy, but this dependence is slight and there is little change in the shape of the angular distribution over the range of energies predicted by the Landau theory for electrons with the same path length in the foil. This statement will only be true for electrons with kinetic energies above 0·5 MeV because, below this energy, the shape of the angular distribution begins to change rapidly with energy. With this reservation, the Landau and Blunck-Leisegang theories will agree in the prediction of the angular distribution of scattered electrons and of the increase in path length, because of the agreement in their prediction of the most probable energy loss.

(*c*) *Choice of Section Length*

The choice of the section length l was governed by the following considerations :

(i) there must be a sufficient number of individual collisions to make valid the equations on which the energy loss and multiple scattering theories are based ;

(ii) the r.m.s. angle of scattering must not be too large since the multiple scattering theories involve a small angle approximation ;

(iii) the mean path length in each section must be close to the section length so that the multiple scattering and energy loss theories may be applied in each section without modification.

The values of section length l finally chosen are set out in Table 1. For heavy elements it is clearly impossible to reconcile the requirement (i) with (ii) and (iii) above.

(*d*) *Results*

The distributions obtained from the Monte Carlo calculations are shown in Figures 1 and 2 (full curves). The method of obtaining smooth curves was to plot the histograms with an interval of 0·01 MeV. Where these points did not

TABLE 1

BEHAVIOUR OF A 1 MEV ELECTRON IN CERTAIN ABSORBERS

Element		Al	Au
Section length, l	(mg/cm²)	5·26	3·68
Average number of collisions (Molière 1948)	Condition (i)	44	34
ξ/I_0Z (Landau 1944)	Condition (i)	2·47	0·24
R.M.S. angle of scattering (from Molière function $f°$)	Condition (ii)	5·3°	9·7°
Mean path length/foil thickness (Yang 1951)	Condition (iii)	1·01	1·04
Most probable energy loss (Landau)	(keV)	4·1	1·8

lie on a smooth curve, larger intervals were chosen and a curve of best fit drawn. The position of the most probable loss was determined by finding the locus of the mean of the abscissae of the two branches of the smoothed curve at equal ordinates and extrapolating to the maximum height. The widths and peak

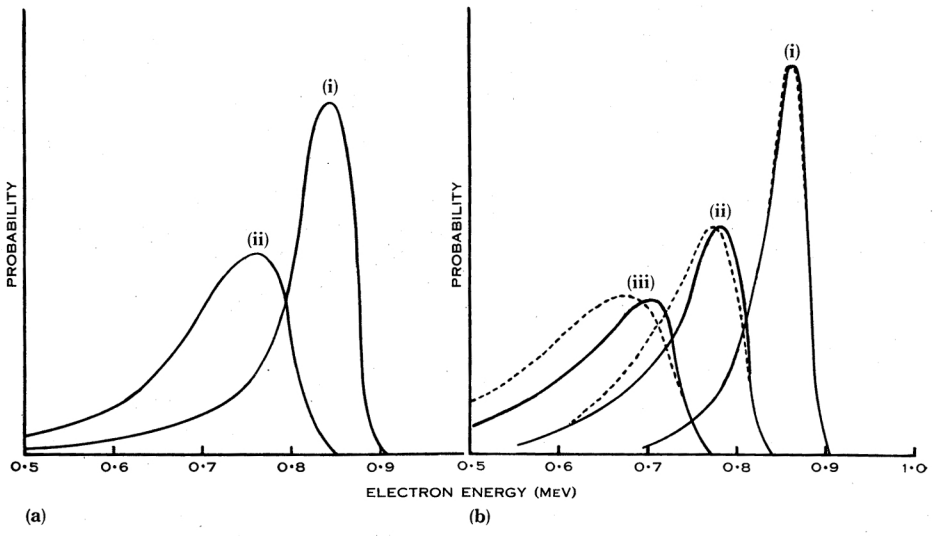

Fig. 1.—Energy distributions of 1 MeV electrons after passing through aluminium; (i) 105 mg/cm², (ii) 158 mg/cm², (iii) 210 mg/cm². (*a*) Monte Carlo (Rossi-Greisen theory) distribution. (*b*) Full curve: Monte Carlo (Molière theory) distribution. Dashed curve: folded (Landau, modified Yang) distribution.

positions of these distributions and the percentage of particles lost are compared in Table 2 with the Landau distribution, and are plotted against absorber thickness in Figure 3.

III. Numerical Integration Method

The effect of multiple scattering on the energy loss distribution has also been calculated by folding the Landau distribution with a path length distribu-

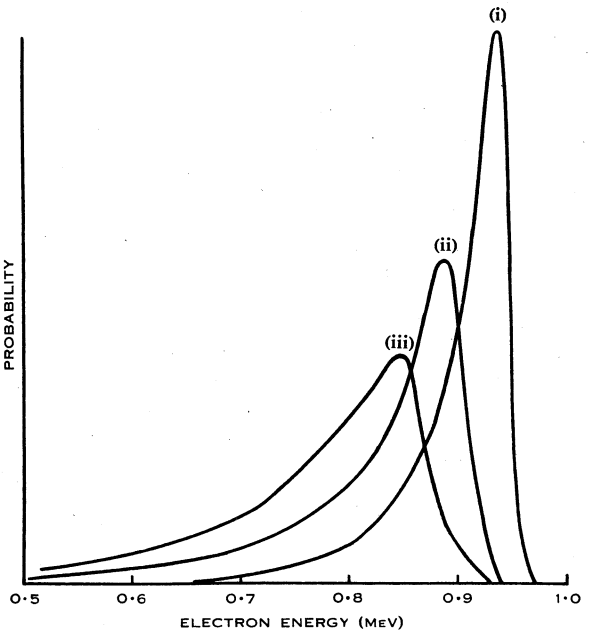

Fig. 2.—Energy distribution of 1 MeV electrons after passing through gold: Monte Carlo (Molière theory); (i) 73 mg/cm², (ii) 110 mg/cm², (iii) 147 mg/cm².

tion. Yang (1951) gives the only available path length distribution, based on the Rossi-Greisen multiple scattering theory. It was found necessary to modify Yang's distribution to give better agreement with the Monte Carlo (Molière

Table 2

Comparison of the Landau theory with the Monte Carlo calculations for 1 MeV electrons at normal incidence

Element and Thickness (mg/cm²)	Most Probable Loss (keV)			Half Height Width (keV)			Loss of Electrons (%)
	Monte Carlo*	Monte Carlo†	Landau	Monte Carlo*	Monte Carlo†	Landau	Monte Carlo*
Al 105	140	155	125	54	75	37	0·2
,, 158	218	236	194	89	134	55	1·5
,, 210	295		264	129		74	8·4
Au 73	63		52	45		21	2·7
,, 110	111		81	75		32	5·0
,, 147	154		113	114		43	8·2

* Monte Carlo: 1000 particles obeying the Molière scattering theory.
† Monte Carlo: 500 particles obeying the Rossi-Greisen scattering theory.

theory) path length distribution. The mean increase of path length according to Yang is nearly twice the mean increase calculated from the theory of Goudsmit and Saunderson (1940) as corrected by Rose (1940). The Goudsmit-Saunderson theory uses a better small angle approximation than the Rossi-Greisen theory. Yang's theory was therefore modified by multiplying his predicted increase of path length (case I) by the factor $\frac{1}{2}$, and the resultant distribution is compared in Figure 4 with the Monte Carlo and the unmodified Yang path length distributions

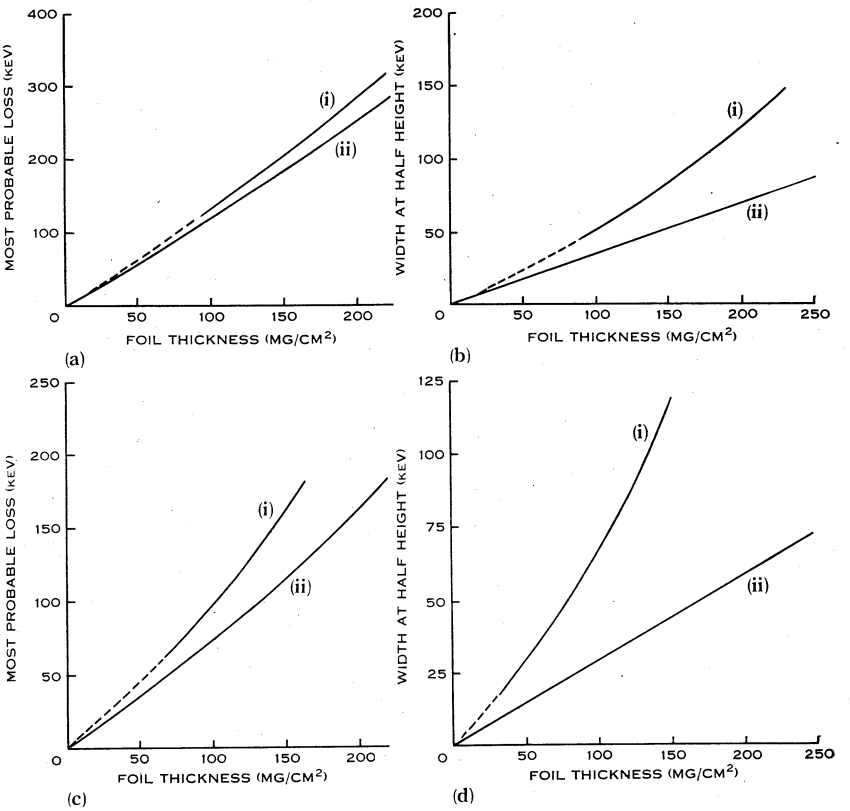

Fig. 3.—1 MeV electrons incident normally on foils; (i) Monte Carlo (Molière theory), (ii) Landau theory. (*a*) Aluminium: most probable energy loss. (*b*) Aluminium: width at half height. (*c*) Gold: most probable energy loss. (*d*) Gold: width at half height.

for a thick aluminium foil. Similar agreement is obtained between the Monte Carlo and modified Yang distributions for a thick gold foil and even better agreement for thinner foils.

The factor $\frac{1}{2}$, giving the correct mean increase of path length, should be valid over a wide range of elements, foil thicknesses, and electron energies, because, although the dependence on these quantities is different for Yang's theory when compared with that of Goudsmit and Saunderson, the difference is in a logarithmic term which varies only slowly.

Figure 1 (*b*) (dashed curves) gives the result of folding the modified Yang path length distributions with the appropriate Landau distributions for 1 MeV electrons incident normally on aluminium, while Table 3 gives a comparison

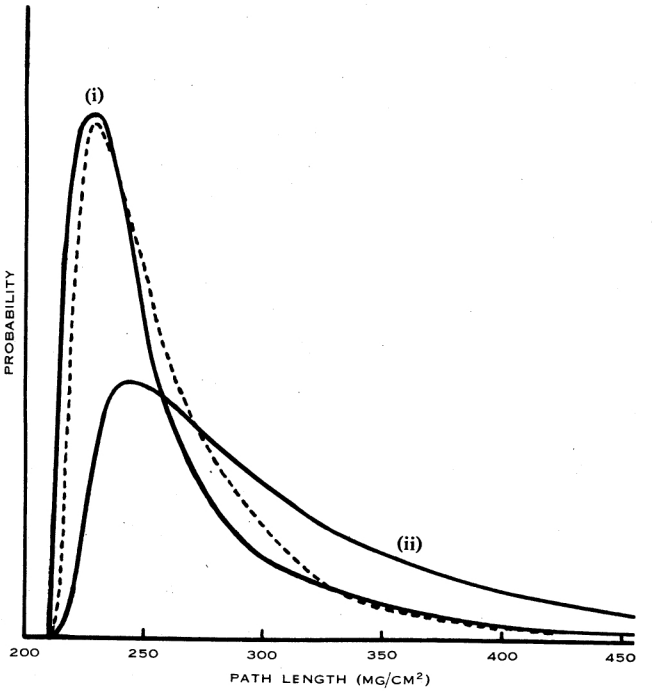

Fig. 4.—Path length distribution of 1 MeV electrons incident normally on 210 mg/cm² of aluminium. (i) Full curve : Monte Carlo (Molière theory) distribution. Dashed curve : modified Yang distribution. (ii) Unmodified Yang distribution.

of widths at half height and most probable losses for the Monte Carlo and folded distributions, and also the estimate of most probable loss obtained by multiplying the Landau value by the ratio of mean path length (modified Yang theory) to

TABLE 3

1 MEV ELECTRONS INCIDENT NORMALLY ON ALUMINIUM

Thickness (mg/cm²)	Most Probable Loss (keV)			Half Height Width (keV)	
	Monte Carlo*	Folded†	Landau $\times \bar{s}/t$‡	Monte Carlo*	Folded†
105	140	140	140	54	54
158	218	228	229	89	101
210	295	322	328	129	170

* Monte Carlo : 1000 particles obeying the Molière scattering theory.

† Folded : the Landau folded with the modified Yang distribution.

‡ $\bar{s}/t$ is the ratio of the mean path length to foil thickness.

foil thickness. It can be shown theoretically that the first effect of multiple scattering is to increase the width at half height and the most probable energy loss by this ratio. Application of this criterion to the Monte Carlo calculations, where peak positions and widths have an error of the order of 3 KeV, leads to thicknesses of 10 sections for both aluminium and gold as the minimum thickness for which multiple scattering is important under these conditions. The agreement with Figure 3 is reasonable.

IV. Application to Experiment

The distributions obtained from these calculations are compared directly with the experiment described in the following paper (McDonell, Hanson, and Wilson 1955) and are further discussed there. However, it is of some interest to apply the criteria developed in these calculations to some experiments to determine the part played by multiple scattering.

Although Chen and Warshaw (1951) claim agreement between experimental and theoretical most probable loss, multiple scattering is important in all cases except the thin beryllium and aluminium foils. Thus the agreement for heavier elements must be somewhat fortuitous. On the other hand multiple scattering is not important in the experiments of West (1953), using various inert gases as absorbers. He finds most probable losses which are on the average 90 per cent. of those predicted by Landau and Blunck-Leisegang, and widths intermediate between these two theories.

In the experiments of Hungerford and Birkhoff (1954) which repeated, more accurately, the work of Kalil and Birkhoff (1953), multiple scattering would be important for all except the thin aluminium and copper foils, if the beam were incident normally. However, the geometry of the source-foil arrangement is such that incident particles are accepted in an angular range from 45 to 55° although only particles emergent at 45° are accepted by the spectrometer. Because of this, multiple scattering will cause significant path length increase even for the thin aluminium and copper foils.

V. Conclusions

Two conclusions can be drawn from the work at this stage. Firstly, the Monte Carlo and the folded distributions for aluminium foils of medium thickness correspond very closely. Therefore, in further calculations of this type, the use of the path length integration method would obviate the considerable labour involved in the Monte Carlo method but retain the same validity. From Figure 1, the energy distribution is sensitive to the multiple scattering theory assumed, and therefore to the assumed path length distribution in the numerical integration method.

Secondly, it appears that the previous workers have not made sufficient allowance for the effect of multiple scattering when comparing experiment with theory. The agreement claimed by these workers for foils of heavier elements indicates that the Landau (or Blunck-Leisegang) most probable losses and the Blunck-Leisegang widths at half height are actually too large, and this is supported by the work of West in cases where multiple scattering is not important.

VI. Acknowledgments

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