THE CONTINUOUS RADIATIVE ABSORPTION CROSS SECTION OF FeXIV AND THE CORONAL TEMPERATURE

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[*Manuscript received September* 17, 1953]

Summary

The continuous radiative absorption cross section of FeXIV was calculated, using a Hartree wave function to evaluate the matrix element for recombination to the ground state. This matrix element was considerably smaller than the value obtained by Hill (1950, 1951) using the hydrogen-like approximation, but the total cross section for recombination was not greatly different. Balancing the rates of recombination and collision ionization in the solar corona then gave a temperature of about 2×10^6 °K, compared with Hill's value of 1×10^6 °K.

I. INTRODUCTION

FeXIV is one of the most important of the highly ionized atoms in the solar corona (Woolley and Allen 1948) and, by balancing the rates of radiative recombination and collision ionization of this ion, one may deduce the electron agitation temperature of the corona. Estimates of these rates were made by Woolley (1947) using classical methods for the ionization cross section and taking a value calculated for OI at 6000 °K, modified by assuming a certain dependence of the cross section on the atomic number and temperature, for the radiative recombination cross section. Improved estimates have since been made by Hill (1951) using wave mechanical methods. Both calculations gave a coronal temperature of close to 10^6 °K. Independent methods of estimating this temperature have given somewhat higher values (see Woolley 1947).

Hill's calculation of the recombination coefficient of FeXIV was made using an appropriate modification of Wessel's formula for the recombination of an electron with a proton. Detailed calculations of recombination coefficients using accurate atomic wave functions have now been made for several neutral atoms and singly ionized atoms, and it appears that the hydrogen-like approximation can sometimes give results seriously in error, as a result of considerable cancellation in the integral for the matrix element. As it seemed likely that such sensitivity of the calculation to the form of the radial wave function would occur, *a fortiori*, for highly charged ions, and as a Hartree field for FeXIV has become available since Hill's paper was written (Gold 1949) an accurate calculation of the recombination coefficient of FeXIV was undertaken.

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II. Radiative Recombination to the Ground State

Quantum theory gives the following expression for the total cross section of an atomic system for radiative recombination

$$Q_R^t(k^2)=\sum_s Q_R^s(k^2)=\frac{128\pi^5 me^2}{3h^2c^3k}\sum_s \nu_{ks}^3 \left| \int \psi_s^* \mathbf{r} \psi_k d\tau \right|^2, \qquad \ldots\ldots (1)$$

ψ_s being the wave function for the bound state, with energy E_S, into which the electron falls, and ψ_k the continuous wave function for the incident electron with momentum $kh/2\pi$ before capture. The frequency of the emitted radiation is ν_{ks}.

If $V(r)$ is the potential of the ion field to which the free electron is captured, then

$$\psi=(2l+1)i^l e^{i\eta_l} k^{-\frac{1}{2}} r^{-1} \sum_{l=0}^{\infty} G_l(r) P_l\,(\cos\theta), \qquad \ldots\ldots\ldots\ldots (2)$$

where G_l is the solution of the differential equation

$$G_l''+[k^2-2V(r)-l(l+1)r^{-2}]G_l=0, \qquad \ldots\ldots\ldots\ldots (3)$$

which vanishes at the origin and has the asymptotic form $k^{-\frac{1}{2}}\sin(kr+\eta_l)$, η_l being a phase shift whose value we shall not require.

The matrix elements sensitive to the form of the wave functions will be those for capture to the ground state of the ion, namely, the state $n=3$, $l=1$. The hydrogen-like approximation used by Hill will be sufficiently accurate for capture to excited states. From the usual selection rule, capture to the ground ($3p$) state can only take place from an s or d state in the continuum, involving the functions G_0 and G_2 respectively. G_0 is the only one of the G's which differs markedly from a Coulomb wave function for $Z=16{\cdot}4$ (corresponding to an effective nuclear charge of $16{\cdot}4e$ in the vicinity of the $3p$ electron), since the $l(l+1)r^{-2}$ term in (3) swamps any deviation of $V(r)$ from the Coulomb field for small r, and at much larger distances the Coulomb field is small. Only $Q_R^{3,1}$, therefore, was calculated using formula (1) with accurate wave functions.

The Hartree field and ground state wave functions have been obtained for FeXIV by Gold (1949), and the required values of $V(r)$ to be used in (3) were found by subtracting the potential $\varphi(r)$ of the $3p$ electron from the Hartree field of FeXIV, where

$$\varphi(r)=\frac{1}{r}\int_0^r \psi^2 d\xi+\int_r^{\infty} \xi^{-1}\psi^2 d\xi, \qquad \ldots\ldots\ldots\ldots (4)$$

$\psi(\xi)$ being the (radial) Hartree wave function (averaged over all angles) for the $3p$ electron. G_0 was then found by numerical integration of (3).

The form of G_0 and ψ_s were found to be such as to cause considerable cancellation in the integrand in (1), and the values obtained for $Q_R^{3,1}$ by adding together the contribution from the s and d states of the continuum were several times smaller than the corresponding values obtained by Hill, as will be seen by comparing the second and third columns in Table 1.

III. Radiative Recombination to the Excited States

The hydrogen-like approximation, as used by Hill, will be sufficiently accurate for capture to the excited states. Hill (1950) has given graphs for various Q_R^n, n being the total quantum number, based on matrix elements calculated by Wessel for $n=1$, 2, 3 and by Bates for not too low energies. At low energies, for $n>3$ and for $(ka)^{-1}>20$ (where $k=2\pi mv/h$ and $a=a_0/Z$), one uses Kramer's classical quantum theory expression

$$Q_R^n=\frac{16e^2h}{3^{3/2}c^3m^2}(ka)^{-4}\frac{1}{n\{n^2+(ka)^{-2}\}^2}. \qquad (5)$$

When allowance is made for the levels which are already filled, we have for the recombination cross section of FeXIV

$$Q_R=Q_R^t-(Q_R^{1,0}+Q_R^{2,0}+Q_R^{2,1}+Q_R^{3,0}). \qquad (6)$$

Values of the various Q_R^n on the right side of (6) were obtained from Hill's graphs, and the necessary correction made from the more accurate values of $Q_R^{3,1}$ found as above. The final value so obtained for Q_R is shown in the last column of Table 1. It is seen that the major contribution to Q_R comes from capture to the higher states, so that Hill's considerable over-estimation of the cross section for capture to the ground state does not have too serious consequences.

Table 1

Recombination Cross Sections

$(ka)^{-1}$	$Q_R^{3,1}$ Werner (10^{-22} cm^2)	$Q_R^{3,1}$ Hill (10^{-22} cm^2)	Q_R^t Hill (10^{-22} cm^2)	Q_R (10^{-22} cm^2)
4	0·07	3·1	55	13
5	0·13	5·4	97	30
6	0·20	8·2	154	55
10	0·73	24·6	537	249
14	1·8	49	1200	635
20	5·1	100	2720	1550
25	9·8	157	4580	2750

IV. Temperature of the Solar Corona

If the peak of the Maxwell distribution came near the plateau of the ionization curve, one might reasonably estimate the temperature of the solar corona by finding the particular energy for which $Q_R=Q_I$. But the peak falls far below the threshold for ionization (below 100 eV compared with a threshold of 376 eV). The temperature of the corona must therefore be obtained by finding the temperature for which $(Q_R)_{\text{Av.}}=(Q_I)_{\text{Av.}}$, where the suffix Av. indicates that the values of Q_R and Q_I must be averaged over a Maxwellian distribution of electron energies.

Using Table 1, $(Q_R)_{\text{Av.}}$ may be obtained for a few temperatures, and, using Hill's ionization curves, $(Q_I)_{\text{Av.}}$ may be obtained for the same temperatures.

The point of intersection of the curves of $(Q_R)_{\mathrm{Av.}}$ and $(Q_I)_{\mathrm{Av.}}$ against temperature gives the required temperature T of the corona.

It is found that about $0 \cdot 4$ of the total value of $(Q_R)_{\mathrm{Av.}}$ comes from the low energy tail of the Maxwell distribution below an energy of 4 eV corresponding to $ka < 1/30$. This is due to the rapid rise in the Q_R^{n}'s towards infinite values as ka tends to zero, so that the value of $Q_R.\mathrm{d}N/\mathrm{d}E$, where $\mathrm{d}N$ is the number of electrons with energies between E and $E+\mathrm{d}E$, when plotted as a function of E, is found to rise again as E falls to low energies. Hill appears to have avoided this embarrassing behaviour by applying a " cut-off " below 4 eV. However, the integrated value of Q_R is found to be finite, and its value over the range $ka=0$ to $1/30$ was obtained by replacing the summation over n in (5) by an integration over n, adjusting the lower limit suitably, and finally integrating over ka. Owing to the high values of Q_R at low energies, a high radiative absorption of electrons in this range will occur, and one might expect an appreciable deviation in the energy distribution from the Maxwell values at low energies, especially as most of the electron-electron collisions will be small-angle, resulting in a slow interchange of electron energies. On the other hand, this effect will be damped out to some extent by the overwhelming predominance of elements other than iron in the solar corona. Such a deviation could exert a considerable change in the value obtained by integrating Q_R.

The values of the averaged Q's obtained are given in Table 2.

TABLE 2

AVERAGE CROSS SECTIONS

T (10^6 °K)	1·0	1·5	2·0	3·0
$(Q_R)_{\mathrm{Av.}}$ (10^{-20} cm²)	3·1		1·4	1·1
$(Q_I)_{\mathrm{Av.}}$ (a) (10^{-20} cm²)		0·66	1·27	2·43
(b) (10^{-20} cm²)		0·53	1·00	1·90

There is some uncertainty in the values of Q_I obtainable from Figure 2 of Hill's paper. The values (*a*) in Table 2 were obtained using the mean of curves 1 and 2, and the values (*b*) using curve 4. These gave the following values for T :

$$\text{Values } (a): \; 2 \cdot 1 \times 10^6 \text{ °K},$$

$$\text{Values } (b): \; 2 \cdot 25 \times 10^6 \text{ °K}.$$

These two values are to be compared with the value $1 \cdot 1 \times 10^6$ °K obtained by Hill, and with the values $2 \cdot 3 \times 10^6$, $3 \cdot 3 \times 10^6$, and $2 \cdot 5$–$6 \cdot 5 \times 10^6$ °K at different heights above the limb, obtained by entirely different methods (see Woolley 1947).

However, the experimental data giving FeXIV as the most abundant of the various ionized states of iron cannot be assumed to apply to the quiescent corona, but will, in fact, be bound to include a proportion (unknown) of coronal " hot spots ". Similarly, the other values of the coronal temperature, obtained by various other methods, would also be some kind of mean between hot spot and quiescent temperatures. The value of the temperature for the quiescent corona, given by the minimum solar radio noise, is about $0 \cdot 5 \times 10^{6}$ °K, which is considerably below the above-mentioned temperatures.

V. Acknowledgment

The author is indebted to Associate Professor C. B. O. Mohr for suggesting this problem and for his continued interest.

VI. References

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