SOME ELECTRON COLLISION CROSS SECTIONS OF CaII

By J. T. JEFFERIES*

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Summary

The cross sections for electron collision excitation of the transitions $4S_{1/2}$—$4P_{1/2}$ and $4P_{3/2}$ in CaII have been calculated using a distorted wave Born approximation. For electrons of energy 4 eV, the calculated values are respectively $6 \cdot 6\pi a_0^2$ and $9 \cdot 9\pi a_0^2$.

I. Introduction

The interpretation of the physical properties of the solar chromosphere and of various phenomena connected with it is assisted by calculations of radiation emitted by model atmospheres, such, for example, as have been performed for hydrogen (Giovanelli 1949 ; Jefferies 1953). To extend these calculations to the important case of CaII the rates of collisional excitation are needed, particularly for transitions between the $4S$ and $4P$ levels, which give rise to the H and K lines. These cross sections are of interest also, as shown by Miyamoto (1953), in assessing the role played by non-coherent scattering in the formation of these lines. In the following, results are given of a calculation of the cross sections for electron excitation of the two transitions $4s^2S_{1/2} \rightarrow 4p^2P_{1/2}$ and $4p^2P_{3/2}$.

II. The Collision Cross Sections

In atomic units the cross section for excitation, by electron collision, of the transition $n \rightarrow n'$ may be written, see, for example, Mott and Massey (1949),

$$\sigma(n \rightarrow n') = \frac{1}{4\pi^2} \frac{k'}{k} \frac{1}{2(2J+1)} \sum_{\substack{m_s, m_{s'} \\ M_J, M_{J'}}} \int |(n|V|n')|^2 \mathrm{d}\Omega, \qquad \ldots\ldots (1)$$

k and k' being the momenta of the incident and scattered electrons, m_s and $m_{s'}$ their spin components, M_J and $M_{J'}$ the components of the total angular momentum of the initial and final states of the ion, and $\mathrm{d}\Omega$ the solid angle into which the electron is scattered. The symbols n and n' involve both the ion and colliding electron. The factor $2(2J+1)$—equal to 4 in the present case—arises from an averaging over the two possible spin orientations of the incident electron and the $2J+1$ values of M_J.

The term $(n|V|n')$ is defined by

$$(n|V|n') = \int \Psi_n V \Psi_{n'}^* \mathrm{d}\tau,$$

* Division of Physics, C.S.I.R.O., University Grounds, Sydney.

V being the interaction energy, and the Ψ's representing wave functions of the complete system, ion and incident electron. The Ψ's should strictly be antisymmetrical combinations in the coordinates (space and spin) of the colliding and ionic electrons. However, for transitions of interest here, it appears (Bates *et al.* 1950) that a better approximation is obtained if exchange effects are neglected, and we accordingly take Ψ to be a simple product of wave functions,

$$\Psi_n = \psi_A(1)\chi_k(2), \quad \Psi^*_{n'} = \psi^*_B(1)\chi^*_{k'}(2),$$

where the ionic and colliding electrons are designated respectively by the numbers 1 and 2.

The $\psi_A(1)$ and $\psi_B(1)$ are taken as appropriate linear combinations—as given for example by Condon and Shortley (1935)—of one-electron wave functions, whose radial components are tabulated by Hartree and Hartree (1935), the angular components being of the central field type.

For the incident electron in the field of the ion, we find

$$\chi_k = r^{-1}k^{-1/2}\sum_{l=0}^{\infty} L_l(k,r)P_l\,(\cos\theta)\begin{cases}\alpha\\ \beta\end{cases}, \quad \ldots\ldots\ldots\ldots \quad (2)$$

where

$$L_l(k,r) = (2l+1)\mathrm{i}^l \mathrm{e}^{\mathrm{i}\eta_l} G_l(r),$$

$G_l(r)$ being the solution of the differential equation

$$G_l''(r) + [k^2 + 2Z_p/r - l(l+1)/r^2]G_l(r) = 0, \quad \ldots\ldots\ldots \quad (3)$$

which goes to zero as r^{l+1} and asymptotes to $k^{-1/2}\sin(kr+\eta_l)$, η_l being a phase whose magnitude is not required for our problem. The symbols α and β represent spin functions and $P_l\,(\cos\theta)$ is the Legendre polynomial. The term $2Z_p/r$ in equation (3) represents the potential of the free electron in the CaII field and can be found from results given by Hartree and Hartree (1935) who tabulate $2Z_p$ for CaI and CaIII. For CaII the mean of these has been used. The wave function of the scattered electron may be written,

$$\chi_{k'} = k'^{-1/2}r^{-1}\sum_{l'=0}^{\infty} L_{l'}(k',r)P_{l'}\,(\cos\Theta)\begin{cases}\alpha\\ \beta\end{cases}, \quad \ldots\ldots\ldots \quad (4)$$

Θ being the angle between the momentum vector of the scattered electron and the radius vector.

Taking the interaction energy V to be $1/r_2 - 1/r_{12}$ and using well-known expansions in terms of associated Legendre polynomials, we may now calculate $(n|V|n')$ and hence the cross section. The results obtained, for $k^2 = 0{\cdot}30$, are

$$\sigma(4s^2S_{1/2} - 4p^2P_{1/2}) = 6{\cdot}6\pi a_0^2,$$

$$\sigma(4s^2S_{1/2} - 4p^2P_{3/2}) = 9{\cdot}9\pi a_0^2.$$

where a_0 is the radius of the first Bohr orbit of hydrogen.

In obtaining these, a sum over the index l of equation (2) has been made. It may be shown (Bohr, Peierls, and Placzek 1949) that, for any value of l, the contribution to the cross section can be no greater than $(2l+1)\pi a_0^2/k^2$. The values obtained here were checked against this condition at each stage, and were found to be compatible with it in all cases ; in only one ($l=1$) did it approach the limiting value. In fact, the results given above are effectively those due to the partial wave with $l=1$.

III. Acknowledgments

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IV. References

Bates, D. R., Fundaminsky, A., Leech, J. W., and Massey, H. S. W. (1950).—*Phil. Trans,* A **243** : 93.

Bohr, N. F., Peierls, R., and Placzek, G. (1949).—Quoted by Mott and Massey (1949).

Condon, E. U., and Shortley, G. H. (1935).—" The Theory of Atomic Spectra." (Cambridge Univ. Press.)

Giovanelli, R. G. (1949).—*Mon. Not. R. Astr. Soc.* **109** : 298.

Hartree, D. R., and Hartree, W. (1935).—*Proc. Roy. Soc.* A **149** : 210.

Jefferies, J. T. (1953).—*Aust. J. Phys.* **6** : 22.

Miyamoto, S. (1953).—*Z. Astrophys.* **31** : 282.

Mott, N. F., and Massey, H. S. W. (1949).—" The Theory of Atomic Collisions." 2nd Ed. (Oxford Univ. Press.)