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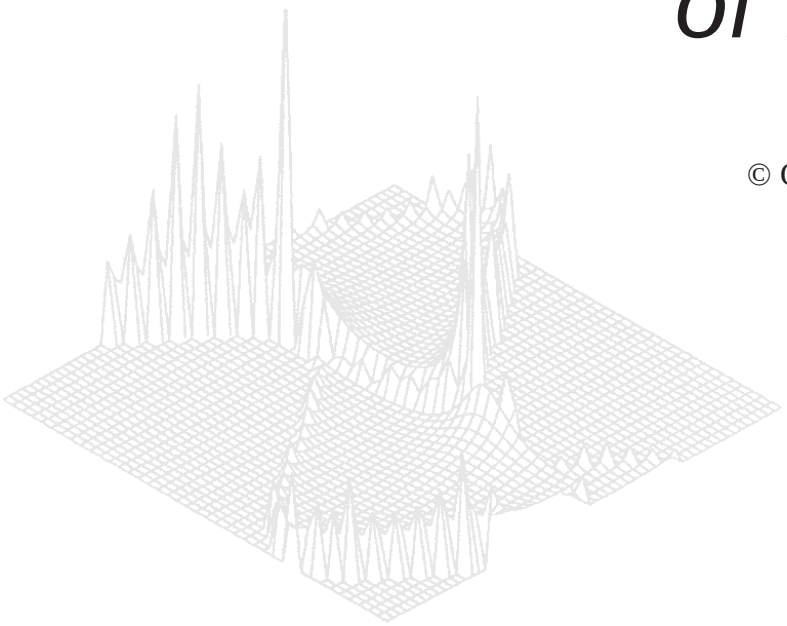
Australian Journal of Physics

Volume 50, 1997



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Aust. J. Phys., 1997, **50**, 13–22

Nonequilibrium Processes in Kaon Condensation*

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Abstract

Relaxation in a kaon condensate proceeds through nonequilibrium weak reactions, which includes the kaon-induced Urca and the modified Urca reactions. The kinetics of a K^- condensate is considered semiclassically starting from a small seed of a condensate with the assumption that the system is in thermal equilibrium. It is shown that the temporal evolution to the K^--condensed phase in the kinetic regime is divided into several characteristic time intervals as a result of the interplay between the two major weak processes. The implications for the dynamical evolution of newly-born neutron stars are briefly discussed.

1. Introduction

The possible existence of kaon condensation in dense matter has been studied extensively, and its implications for astrophysical phenomena and heavy-ion collisions have been widely discussed [1]. It has been shown that the S-wave kaon condensation can be considered almost model-independently on the basis of $SU(3)_L \times SU(3)_R$ current algebra and PCAC [2, 3]. In this framework, the classical K^- field is written as $\langle K^-|\hat{K}^-|K^-\rangle = f/\sqrt{2}\cdot\sin\theta\exp(-i\mu_K t)$ with the meson decay constant f ($\simeq$93 MeV), the kaon chemical potential μ_K, and the chiral angle θ, which represents the order parameter of the condensate.

In the kaon condensed phase realized inside a neutron star, the weak reactions play an essential role in both the ground-state properties and its thermal evolution: In stable neutron star matter, net strangeness is produced through the weak reaction (K), $n+n \rightleftharpoons n+p+K^-$, and the K^--condensed phase is realized by satisfying the chemical equilibrium for this weak process [4]. Weak reactions are also responsible for some *nonequilibrium* processes to the K^--condensed neutron stars. One of the relevant weak reactions is the kaon-induced Urca reaction (KU), $n(p) \to n(p) + e^- + \bar{\nu}_e$, $n(p) + e^- \to n(p) + \nu_e$ [2], and another is the modified Urca process (MU), $n + n \to n + p + e^- + \bar{\nu}_e$, $n + p + e^- \to n + n + \nu_e$ [5, 6]. For example, in newly born neutron stars just after supernova explosions, the

* Refereed paper based on a contribution to the Japan–Australia Workshop on Quarks, Hadrons and Nuclei held at the Institute for Theoretical Physics, University of Adelaide, in November 1995.

10.1071/PH96040 0004-9506/97/010013$05.00

K^--condensed phase may develop from normal neutron-star matter through the *relaxation* process due to these weak reactions: When the baryon number density of the matter n_B exceeds the onset density of the K^- condensed phase n_B^c, it takes some time to reach equilibrium because the characteristic time scales for the weak reactions are longer than those for the strong and electromagnetic interactions and the gravitational collapse ($\sim 10^{-3}$ s). In contrast, as the classical K^- field is controlled by the strong kaon–nucleon (KN) and kaon–kaon (KK) interactions, it responds instantaneously to the change in the chemical composition driven by the weak reactions. Hence the equation of state (EOS) of the K^--condensed phase may be considered adiabatically.

The weak reactions, KU and MU, are also important in connection with the thermal evolution of neutron stars. Especially, the reaction KU has been studied as an exotic cooling mechanism associated with the K^- condensate [2, 3].*

The situation for the nonequilibrium processes in the kaon condensation is similar to that for strange quark matter, where the strangeness production occurs via the nonleptonic weak reactions, $u + d \rightleftharpoons s + u$, together with the semileptonic reactions, $d(s) \to u + e^- + \bar{\nu}_e$, $u + e^- \to d(s) + \nu_e$, leading to chemical equilibrium [7]. The nonequilibrium processes in kaon condensation also has some relevance to the recent studies of the growth of a Bose condensate in condensed-matter physics [8]. Here we investigate how normal matter is converted to the K^--condensed phase through these nonequilibrium processes.

2. Formulation

(2a) Kinetics of the Condensate

Consider uniform normal neutron-star matter ($\theta = 0$) with a baryon number density n_B ($> n_B^c$) which is out of chemical equilibrium. The dynamical evolution of the system toward the equilibrium K^--condensed phase then follows through the nonequilibrium weak processes, K, KU and MU.

In general, the relaxation process in a Bose condensate can be divided into three sequential regimes [8]: (I) the kinetic regime, where the Bose distribution function is restructured; (II) the coherent regime, where the Bose condensate is formed through fully quantum effects such as nucleation; and (III) the kinetic regime, where the growth of the Bose condensate proceeds, finally reaching chemical equilibrium. In the second regime (II), the thermal kaons, produced via the weak process (K) and other weak processes are converted into condensed kaons. However, the time required for the nucleation of the condensate in regime (II) may be negligible as compared with those for (I) and (III), where the relevant time scale is determined by the weak reactions. In the following formulation, therefore, we do not pursue the detailed dynamics of conversion of thermal kaons into a condensate in regime (II). Instead, we assume, throughout the relaxation process, that the system stays in thermal equilibrium via the

* It has been shown that the direct Urca process (D), $n \to p + e^- + \bar{\nu}_e$, $p + e^- \to n + \nu_e$ in the K^--condensed phase would be another unique process in the K^- condensate [3]. In the case where the process D is operative, the process MU should be replaced by D for cooling of neutron stars. However, whether the direct Urca process is allowed or not is determined by the symmetry energy, whose density dependence is not well known. Here we consider the case where the process MU is responsible for the neutrino emission.

strong and electromagnetic interactions. Then, the distribution function for the kaon, for example, is written as

$$f_K(\mathbf{k},t) = (2\pi)^3\zeta_K(t)\delta^{(3)}(\mathbf{k}) + \frac{1}{e^{(\omega_- - \mu_K)/T}-1} - \frac{1}{e^{(\omega_+ + \mu_K)/T}-1} \ , \qquad (1)$$

where the first term is the condensed part, and the second and third terms are the Bose–Einstein distribution functions of the K^- and K^+ mesons, respectively. With this form, one can see that the condensate appears automatically when the number of the thermal kaons [corresponding to the second and third terms of Eq. (1)] are saturated. Thus, we concentrate on the growth of the K^- condensate in the kinetic regime.

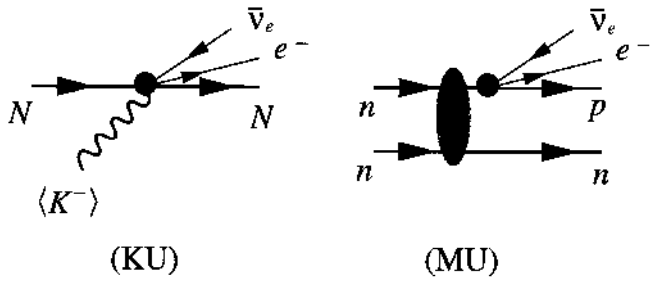

Fig. 1. Lowest-order diagrams for the kaon-induced Urca reaction (KU), $n(p) \to n(p) + e^- + \bar{\nu}_e$, $n(p) + e^- \to n(p) + \nu_e$, and the modified Urca one (MU), $n + n \to n + p + e^- + \bar{\nu}_e$, $n + p + e^- \to n + n + \nu_e$. In both figures, only forward processes are shown.

(2b) Rate Equations

The temporal changes of the number densities of the particle species are determined by the rate equations, which can be derived from the semi-classical transport equation. For the electrons, we get

$$\begin{aligned} dn_e(t)/dt = {} & \Gamma^{(KU)}(\xi^{(KU)}(t)) - \Gamma^{(KU)}(-\xi^{(KU)}(t)) \\ & + \Gamma^{(MU)}(\xi^{(MU)}(t)) - \Gamma^{(MU)}(-\xi^{(MU)}(t)) \end{aligned} \qquad (2)$$

with $n_e(t) = \mu_e(t)^3/3\pi^2$. In Eq. (2), $\Gamma^{(\alpha)}(u)$ and $\Gamma^{(\alpha)}(-u)$ (for $\alpha =$ KU, MU) are the forward and backward reaction rates, respectively, for the processes KU and MU in the kaon-condensed phase, and $\xi^{(KU)}(t) \equiv [\mu_e(t) - \mu_K(t)]/T$ and $\xi^{(MU)}(t) \equiv [\mu_p(t) + \mu_e(t) - \mu_n(t)]/T$ are the chemical potential differences normalized by the temperature T, which are the measures of the deviations from chemical equilibrium. In the case $\xi^{(\alpha)}(t) = 0$, the forward and backward reaction

rates are equal, and the system is in chemical equilibrium. The deviations from the equilibrium number densities for other particles are related via

$$\begin{aligned}
&\delta^{(KU)}n_K(t) = -\delta^{(KU)}n_e(t),\ \ \delta^{(KU)}n_p(t) = \delta^{(KU)}n_n(t) = 0,\\
&\delta^{(MU)}n_K(t) = 0,\ \ \delta^{(MU)}n_p(t) = -\delta^{(MU)}n_n(t) = \delta^{(MU)}n_e(t),
\end{aligned} \tag{3}$$

where $\delta^{(\alpha)}n_i(t)$ is the deviation of the number density of the particle species i $(= p, n, K^-, e^-)$ due to process α (cf. Fig. 1). The number density of the kaons $n_K(t)$ is identified with the negative strangeness density, or equivalently, the electromagnetic charge density of the kaons: $n_K(t) = \zeta_K(t) + n_K^T(t)$, with the condensed part

$$\zeta_K(t) \equiv \langle K^-|\hat{S}|K^-\rangle = \mu_K(t) f^2 \sin^2\theta(t) + [1-\cos\theta(t)][n_p(t) + \tfrac{1}{2}n_n(t)] \tag{4}$$

and the thermal part $n_K^T(t)$. With this identification for $n_K(t)$, the charge neutrality condition is written as $n_p(t) = n_K(t) + n_e(t)$, which is guaranteed during the nonequilibrium processes via Eq. (3).

The $\Gamma^{(\alpha)}(u)$ are obtained from the matrix elements and the phase-space integrals. The relevant weak Hamiltonian is given by $\widetilde{H}_{\rm W} = \dfrac{G_{\rm W}}{\sqrt{2}}\widetilde{J}_{\rm h}^{\mu} l_{\mu}$ +h.c., with the effective hadronic current

$$\begin{aligned}
\widetilde{J}_{\rm h}^{\mu} &= \hat{U}_K^{-1} J_{\rm h}^{\mu} \hat{U}_K \\
&= \cos\theta_c\big\{(V^{\mu}_{1+i2} - A^{\mu}_{1+i2})\cos(\theta/2) + i(V^{\mu}_{6-i7} - A^{\mu}_{6-i7})\sin(\theta/2)\big\} \\
&\quad + \sin\theta_c\big\{(V_4^{\mu} - A_4^{\mu}) + i\cos\theta(V_5^{\mu} - A_5^{\mu}) - \frac{i}{2}\sin\theta(V_3^{\mu} - A_3^{\mu} + \sqrt{3}(V_8^{\mu} - A_8^{\mu}))\big\},
\end{aligned} \tag{5}$$

where $\hat{U}_K$ is the unitary operator which generates the kaon-condensed state $|K^-\rangle$ from the normal state $|x\rangle$ by a chiral rotation: $|K^-\rangle = \hat{U}_K|x\rangle$. In Eq. (5), θ_c $(\simeq 0{\cdot}24)$ is the Cabibbo angle, and V_a^{μ} (A_a^{μ}) is the vector (axial vector) current [2]. Process KU is mediated by the last term (the 'commutator' contribution) in Eq. (5), and process MU by the first term proportional to $\cos\theta_c$.

For the KU reaction, we get

$$\Gamma^{(KU)}(\xi^{(KU)}(t)) = (6{\cdot}6\times 10^{29})\left(\frac{m_N^*}{m_N}\right)^2 \frac{\mu_e(t)}{m_\pi}\sin^2\theta(t) T_9^5 I_2(\xi^{(KU)}(t))\ \ ({\rm cm}^{-3}\cdot{\rm s}^{-1}) \tag{6}$$

where m_N^*/m_N is the effective to the free nucleon mass ratio, m_π the pion mass, T_9 the temperature in units of 10^9 K, and $I_2(u) \equiv \int_0^\infty dx x^2[\pi^2 + (x+u)^2]/[1+\exp(x+u)]$.

For the MU reaction, we refer to Friman and Maxwell's result [5, 6]. Noting that the matrix elements are slightly modified in the presence of a K^--condensate

by an additional factor of $\cos^2(\theta/2)$ coming from the first term of Eq. (5), we get

$$\Gamma^{(MU)}(\xi^{(MU)}(t)) = (5{\cdot}9\times10^{23})\left(\frac{n_e(t)}{n_0}\right)^{\frac{1}{3}}\cos^2(\theta(t)/2)T_9^7 J_2(\xi^{(MU)}(t))\ \ (\mathrm{cm}^{-3}\cdot\mathrm{s}^{-1}) \tag{7}$$

with $J_2(u)\equiv\int_0^\infty dx x^2[9\pi^4+10\pi^2(x+u)^2+(x+u)^4]/[1+\exp(x+u)]$, and n_0 ($=0{\cdot}17$ fm^{-3}) is the standard nuclear density.

The dynamics of the K^--condensate is implemented by the classical field equation for the order parameter (chiral angle) θ. Accordingly, $\theta(t)$ may change adiabatically from zero to the equilibrium value θ^{eq}, following the extremum condition of the energy density of the K^--condensed phase $\mathcal{E}$ with respect to θ; $\partial\mathcal{E}/\partial\theta=0$. It reads

$$\mu_K(t)^2\cos\theta(t) - m_K^2 + \frac{1}{f^2}\{n_B\Sigma_{\mathrm{KN}} + \mu_K(t)[n_p(t)+\tfrac{1}{2}n_n(t)]\} = 0\ . \tag{8}$$

The first and second terms in the curly bracket of Eq. (8) stem from the S wave K^-N interactions given by the KN sigma term, Σ_{KN}, and the Tomozawa–Weinberg term, respectively [3].

3. Numerical Results and Discussion

In the actual calculation, we make some further simplifications: (i) The temperature T is assumed to be sufficiently low and is taken to be constant during the relaxation process. Thermal effects are assumed to be insignificant except for the initial stage of the relaxation process, where a small K^- condensate may be formed in the normal phase due to thermal fluctuations. Thus, we neglect the thermal part $n_K^T(t)$ in the strangeness number density, and a small seed of a condensate, $\theta^0\equiv\theta(t=0)=10^{-4}\sim10^{-3}$, is put in by hand initially.* (ii) For the equation of state (EOS), we start with the uniform normal neutron-star matter in beta equilibrium with the fixed baryon number density n_B ($>n_B^c$). For this EOS, we take into account the potential contribution to the symmetry energy $V_{\mathrm{sym}}(n_B)$, for which we use the results in Ref. [9]. In this case, the initial proton mixing ratio $x^0\equiv n_p^0/n_B=0{\cdot}062$ ($\equiv x_N^{\mathrm{eq}}$) for $n_B=0{\cdot}50\mathrm{fm}^{-3}$, and the onset density for the K^- condensation is given as $n_B^c=0{\cdot}48\mathrm{fm}^{-3}$ for $\Sigma_{KN}=400$ MeV. We take the initial kaon chemical potential μ_K^0 as the lowest excitation energy of K^-, which is obtained from Eq. (8) by taking the limit of $\theta\to0$.

Figs 2, 3 and 4 show, respectively, the time dependences of the chiral angle θ, the normalized chemical potential difference $\xi^{(\alpha)}(t)$, and the forward and backward reaction rates, $\Gamma^{(\alpha)}(\xi^{(\alpha)}(t))$ and $\Gamma^{(\alpha)}(-\xi^{(\alpha)}(t))$. The temporal behaviour in the number density of each constituent particle species is shown in Fig. 5. The parameters used are $n_B=0{\cdot}50$ fm^{-3}, $T=1\times10^{10}$ K and $\Sigma_{KN}=400$ MeV. One observes that θ takes on a small value ($\sim\theta^0$) until τ_1 (several tens of a millisecond), after which it increases rapidly. Therefore, for $t\le\tau_1$, the reaction rate for KU is small, because it is proportional to $\sin^2\theta$ (Eq. (6)). This is so even though the deviation from chemical equilibrium due to the reaction KU is large ($\xi^{(KU)}(t)\sim30$

* Hereafter, the superscript '0' denotes the value at $t=0$.

from Fig. 3). The reaction rate for MU is also small because we start with beta equilibrium in the normal phase i.e. $\xi^{(MU)}(0) \simeq 0$ (cf. Figs 3 and 4). The time scale τ_1 is estimated as follows: For $|u| \gg 1$, the function $I_2(u) - I_2(-u)$ has an asymptotic form, $\simeq -u^5/30$. For $|\xi^{KU}(t)| \gg 1$, the total reaction rate for KU is then written as $\Gamma^{(KU)}(\xi^{(KU)}(t)) - \Gamma^{(KU)}(-\xi^{(KU)}(t)) \sim -\dfrac{\alpha(t)}{30}\sin^2\theta(t)[T\xi^{(KU)}(t)]^5$, where $\alpha(t)$ is a factor defined as $\Gamma^{(KU)}(u) = \alpha(t)\sin^2\theta(t)T^5 I_2(u)$ in Eq. (6). On the other hand, for $\theta(t) \ll 1$, the strangeness density is approximated as $n_K(t) = \{\mu_K^0 f^2 + n_B(1+x^0)/4\}\theta^2(t) + O(\theta(t)^4)$. The rate equation for $n_K(t)$ is thus expressed in terms of $\theta(t)$ as $d\theta^2(t)/dt \sim A\theta^2(t)$ with

$$A \equiv \alpha^0(\mu_e^0 - \mu_K^0)^5/\{30[\mu_K^0 f^2 + n_B(1+x^0)/4]\} \ . \tag{9}$$

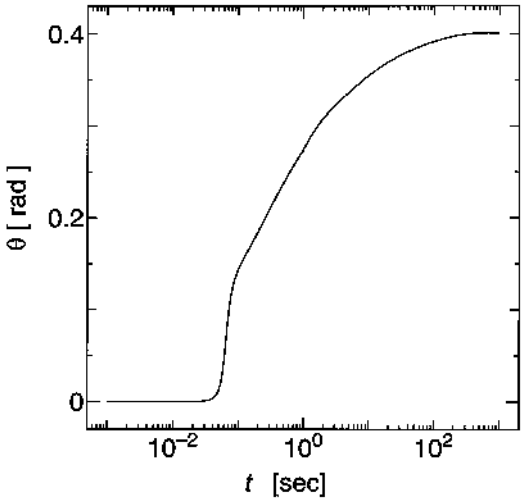

Fig. 2. Chiral angle θ as a function of time for $n_B = 0{\cdot}50$ fm^{-3}, $T = 1{\times}10^{10}$ K and $\Sigma_{KN} = 400$ MeV.

Taking $\theta(t) = 0{\cdot}01$ where the reaction rate for KU becomes significantly large, we can estimate the characteristic time τ_1 as $\tau_1 \sim 2/A \cdot \ln(1/10^2\theta^0)$, after which $\theta(t)$ grows rapidly. It is to be noted that τ_1 depends on the initial chiral angle of the condensate θ^0, which introduces an ambiguity in these results. Nevertheless, the dependence of τ_1 on θ^0 is logarithmic, so that this time scale does not change much within the range of our choice, $\theta^0 = 10^{-4} \sim 10^{-3}$. However, it is necessary to justify the value for θ^0 by taking into account the thermal part of the strangeness number density $n_K^T(t)$ explicitly. The timescale τ_1 is independent of temperature, provided the initial deviation for KU is large, i.e. $|\xi^{(KU)}(t)| \gg 1$. This feature stems from the characteristic form of the total reaction rate for $|\xi^{(KU)}(t)| \gg 1$. The magnitude of τ_1 depends rather sensitively on density through the initial chemical potential difference due to reaction KU: $\tau_1 \propto (\mu_e^0 - \mu_K^0)^{-5}$. For example, $\mu_e^0 - \mu_K^0 = 0{\cdot}14$ fm^{-1} $(1{\cdot}35$ fm$^{-1})$ for $n_B = 3n_0$ $(4n_0)$, and one gets $\tau_1(4n_0)/\tau_1(3n_0) \sim 1{\cdot}2 \times 10^{-5}$.

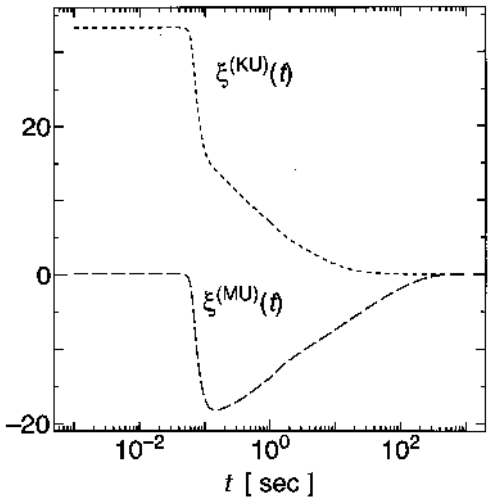

Fig. 3. Normalized chemical potential differences $\xi^{(KU)}(t)$ (dotted curve) and $\xi^{(MU)}(t)$ (dashed curve) as a function of time.

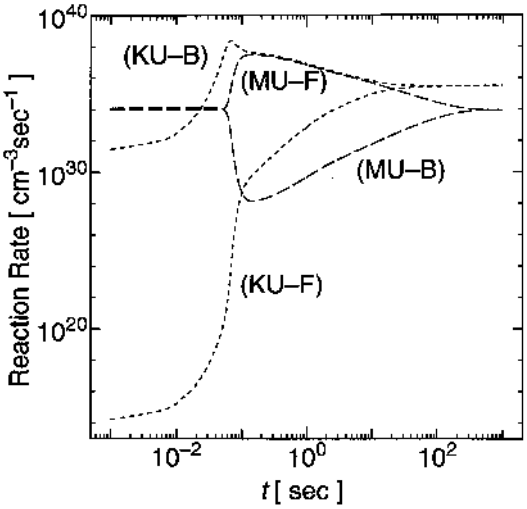

Fig. 4. Temporal behaviour of the reaction rates in units of $\mathrm{cm}^{-3}\,\mathrm{s}^{-1}$. KU-F and KU-B, shown by the dotted curves, denote the forward and the backward processes of the kaon-induced Urca reaction; MU-F and MU-B, shown by the dashed curves, denote the forward and the backward processes of the modified Urca reaction, respectively.

Immediately after $t \sim \tau_1$, the reaction rate for the backward KU reaction (denoted by KU-B in Fig. 4) becomes dominant, such that the K^--condensate grows significantly to reach a value $\theta(t) \sim 0{\cdot}1$. This occurs within $t = 0{\cdot}1\,\mathrm{s}$ ($\equiv \tau_2$), as seen in Figs 2 and 4. One can also see in Fig. 5 that the strangeness density $n_K(t)$ begins to increase at $t = \tau_1$.

In the time interval $\tau_1 \leq t \leq \tau_2$, when the condensate develops via the reaction $n(p) + e^- \rightarrow n(p) + \langle K^- \rangle + \nu_e$, the electron number density, or its chemical potential decreases (cf. Fig. 5). As a result, $\xi^{(KU)}(t)$ decreases toward equilibrium due to reaction KU, as seen in Fig. 3. While $\xi^{(MU)}(t)$, which starts from almost zero initially, takes on a negative value (~ -20) due to the decrease in μ_e (cf. Fig. 3). For $\tau_2 \leq t \leq \tau_3$ ($\sim$10 s), this deviation induces the MU reaction. In this time interval, the two processes KU and MU compete. Especially, the forward reaction rate MU, $n + n \rightarrow n + p + e^- + \bar{\nu}_e$, (denoted by MU-F in Fig. 4) becomes as large as that for the backward reaction KU-B, which tends to counterbalance the change in the electron number density. As a result, the magnitude of $\xi^{(MU)}(t)$ decreases monotonically toward equilibrium as well as $\xi^{(KU)}(t)$. There is a gradual net decrease in the electron number density, while the proton number density increases gradually due to the process MU-F (see Fig. 5).

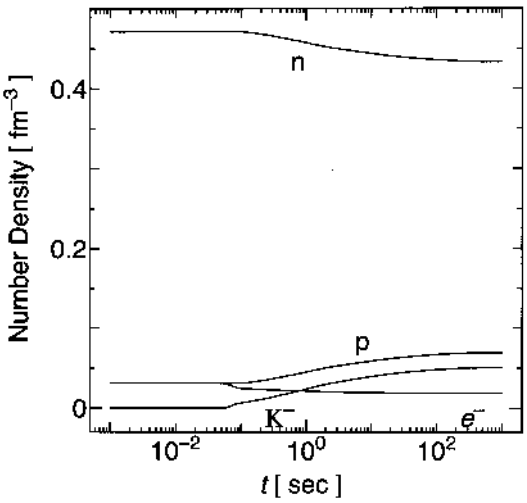

Fig. 5. Temporal behaviour of the proton, neutron, kaon (strangeness) and electron number densities in units of fm^{-3}.

At the end of this competition period $t \sim \tau_3$, the values of $\xi^{(KU)}(t)$ and $\xi^{(MU)}(t)$ are not large any more. In such a case, the reaction rates have a complicated temperature dependence through the functions $I_2(\xi^{(KU)}(t))$ and $J_2(\xi^{(MU)}(t))$, so that the relaxation in this region will proceed in a temperature-dependent manner.

From Fig. 4, one sees that, at around τ_3, the reaction rates of the processes KU-F and KU-B become equal, i.e. the system has reached chemical equilibrium due to reaction KU. Furthermore, one finds that $\xi^{(KU)}(t)$ tends to almost zero (cf. Fig. 3). At the same time, θ reaches the equilibrium value, $\theta^{\rm eq} = 0 \cdot 40$. The EOS is determined mainly by the strength of the condensate, i.e. the magnitude of θ. Hence the EOS is mostly settled to an equilibrium value at $t \sim \tau_3$.

In the last stage of the relaxation process ($t \geq \tau_3$), both $\xi^{(KU)}(t)$ and $\xi^{(MU)}(t)$ become small. When $|\xi^{(KU)}(t)| < 1$ and $|\xi^{(MU)}(t)| < 1$, the total reaction rates, $\Gamma^{(\alpha)}(\xi^{(\alpha)}(t)) - \Gamma^{(\alpha)}(-\xi^{(\alpha)}(t))$, may be expanded up to the first order in $\xi^{(\alpha)}(t)$. Then the rate equations (2) can be linearized and an analytic treatment is possible.

4. Summary and Concluding Remarks

We have investigated the relaxation process in a kaon condensate through the nonequilibrium weak reactions—the kaon-induced Urca reaction and the modified Urca reaction. The kinetics of the K^- condensate has been considered semiclassically starting from a small seed of a condensate, under the assumption that the system is in thermal equilibrium. Dynamical evolution of the K^--condensate in the kinetic regime is divided into the following characteristic time intervals: (1) Stagnation of the condensate, where a small seed of a condensate grows very slowly to a non-negligible value, due to a small reaction rate for the process KU. (2) A rapid rise of the condensate, where the process KU dominates in the relaxation process. (3) Competition of the two processes KU and MU, where the change in the electron number density due to the reaction KU induces the reaction MU. Hereafter, the system gradually approaches chemical equilibrium, and (4) the asymptotic region, where the system is near equilibrium. The relaxation time in this asymptotic region is obtained analytically.

We have not considered the coherent regime explicitly and assumed that the system is in thermal equilibrium throughout the relaxation process. The dynamics in the coherent regime, where full quantum effects should be taken into account, is another interesting issue, where the conversion mechanisms of thermal kaons into a condensate must be studied. As a substitute for the dynamics in this regime, we began with a small seed of a condensate, θ^0. On the assumption of thermal equilibrium, however, the strangeness number density is given by the use of Eq. (1), so that we can take into account implicitly the conversion of the thermal part of the strangeness number density to that of a condensate together with the kinetic equations.

As for the kaon-condensed EOS, we have adopted the zero-temperature form. We have also used the reaction rates in the low temperature limit, where the fermions are fully degenerate. For $T \leq$ several tens of MeV, these approximations are valid. However, for a neutron star just born in a supernova, the initial temperature may be as high as $T \geq$ several tens of MeV. Hence, if we pursue the dynamical evolution of a neutron star beginning with such a high temperature, we need to use the kaon-condensed EOS at finite temperatures, and treat the thermal effects in a more realistic manner. Specifically, the proto-neutron star matter at $T \sim$ several tens of MeV will not be transparent to neutrinos. In such a case, the trapping of neutrinos in matter and the neutrino degeneracy must be taken into account [7].

During the relaxation process, the temperature is assumed to be constant. In reality, however, the energy release through various nonequilibrium processes may occur, which could heat up the system. Therefore, the effects such as the change in the temperature of the system due to energy release may play a crucial role in the thermal and dynamical evolution of neutron stars [7].

Recently, a scenario for the production of low-mass black holes was presented by taking advantage of the very soft equation of state of the kaon condensate [10]. According to this scenario, the maximum neutron star mass is at most $\sim 1{\cdot}5 M_\odot$, and a larger mass leads to a black hole. It should be mentioned that their argument is based on the static EOS for kaon condensation. However, the dynamical processes discussed here might have crucial effects. For example, a neutron star, which would finally collapse into a black hole, may evolve as a

stable configuration in the course of the relaxation process. In addition, how the time scale for such a relaxation process affects the stability of a kaon-condensed neutron star presents an interesting issue in supernova/neutron star physics.

Acknowledgments

Two of the authors (T.M. and T.T.) would like to express their sincere thanks to Professor A. W. Thomas, Dr A. G. Williams and Dr K. Saito, who organized the workshop, "Quarks, Hadrons and Nuclei", and for the warm hospitality at Adelaide. The authors also thank Professor R. Tamagaki and Professor T. Takatsuka, Dr H. Fujii, and Mr T. Tanaka for useful discussions and comments. N.I. wishes to thank Professor C. G. Montgomery for comments on the manuscript and NSF for partial support through Grant No. PHY-90–08475.

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