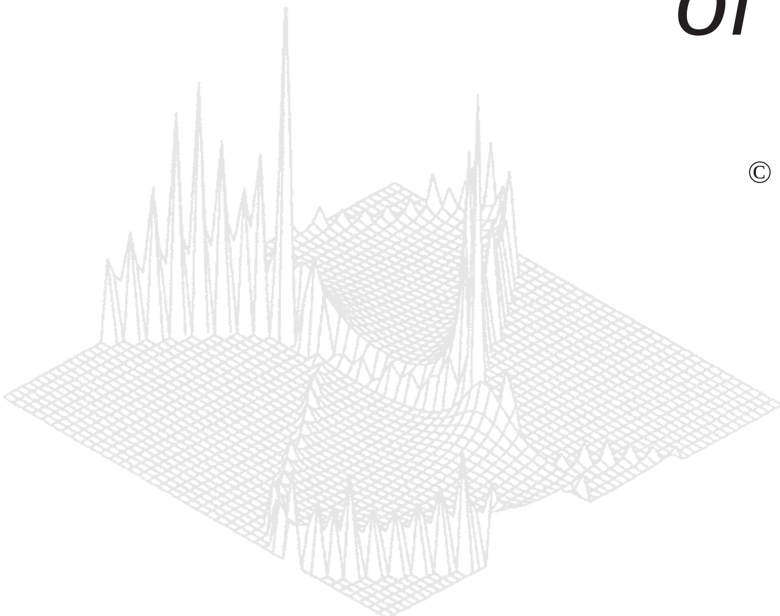
CSIRO PUBLISHING

Australian Journal of Physics

Volume 50, 1997



A journal for the publication of
original research in all branches of physics

www.publish.csiro.au/journals/ajp

All enquiries and manuscripts should be directed to
Australian Journal of Physics
CSIRO PUBLISHING
PO Box 1139 (150 Oxford St)
Collingwood
Vic. 3066
Australia

Telephone: 61 3 9662 7626
Facsimile: 61 3 9662 7611
Email: peter.robertson@publish.csiro.au

CSIRO AUSTRALIA

Published by **CSIRO** PUBLISHING
for CSIRO Australia and
the Australian Academy of Science

Academy of Science

Aust. J. Phys., 1997, **50**, 211–20

Algebraic Method for Large-N_c QCD*

Atsushi Hosaka[A] *and Niels R. Walet*[B]

[A] Numazu College of Technology, 3600 Ooka, Numazu,
Shizuoka, 410 Japan.
[B] Department of Physics, UMIST, PO Box 88,
Manchester, M60 1QD, United Kingdom.

Abstract

We discuss an algebraic method for large-N_c baryon structure. In addition to the widely used quark representation for the large-N_c algebra, we consider several other representations that reflect important dynamical effects. The axial vector coupling constants both in the isoscalar (flavour-singlet) and isovector sectors are discussed in detail. Importance of the pion degrees of freedom is pointed out.

1. Introduction

In the last decade, hadron physics, in particular, baryon structure at low energies has been studied extensively using effective models. They include quark models (relativistic and non-relativistic ones), the Skyrme model, their hybrid models (chiral bag, chiral quark models, ...), the NJL model etc. Many of these models are able to describe nucleon properties almost at the same accuracy, which is about typically at the 30% level, although their model setups look quite different. For instance, the quark model and the Skyrme model appear to be built upon totally different ground.

However, there is one common aspect which is shared by all these models: that is, the group structure for spin and isospin (for $N_f = 2$) symmetry. This is so because hadron spectra respect spin and isospin symmetry. It is then natural to expect that the success of these models would be just a consequence of the same underlying group structure. In fact, there was an attempt to relate the quark and the Skyrme models by regarding them as different limits of the same group representation [1, 2, 3]. For example, nucleon matrix elements of $\sigma\tau$ have been calculated using the so-called quark representation and shown to contain the same group theoretical factor [3]:

$$g_A \sim \frac{N_c}{3}\left(1 + \frac{2}{N_c}\right). \tag{1}$$

* Refereed paper based on a contribution to the Japan–Australia Workshop on Quarks, Hadrons and Nuclei held at the Institute for Theoretical Physics, University of Adelaide, in November 1995.

10.1071/PH96031 0004-9506/97/010211$05.00

This reproduces the famous quark model factor $\frac{5}{3}$ when $N_c = 3$ and the Skyrme model result $g_A \sim \mathcal{O}(N_c)$ when $N_c \to \infty$. Such an algebraic method should in principle be useful in making model independent predictions and in deriving relations which become exact in the large-N_c limit. In some cases, it is even possible to calculate higher $1/N_c$ corrections unambiguously. These are the reasons that there have been many papers recently that have been concerned with baryon properties based on the large-N_c algebra [4, 5, 6, 7, 8]. In fact, motivated by the Skyrmion quantization at finite N_c, Amado *et al.* independently studied essentially the same algebraic method [9, 10].

In this paper we consider a group theoretical aspect for large-N_c baryons, whose algebra is dictated by a contracted SU(4) for the spin and isospin symmetry. We investigate several representations in addition to the quark representation which have been commonly used in the literature [1, 6]. The reasons are twofold:

(1) In contrast to the widespread belief that the quark representation produces both the Skyrme model and the quark model matrix elements, we show explicitly that there is an example in which this is not the case. That is the isoscalar axial vector coupling constant $g_A^{(0)}$, the nucleon spin fraction carried by quarks.

(2) The quark representation is essentially an algebraic realization of the non-relativistic quark model. We consider that there are important dynamical effects missing in the quark representation, which cannot be ignored for a realistic description of the nucleon. They are the relativistic effect of the quarks and the pion-cloud effect. We compute the axial vector coupling constants to show the role of these two effects.

We organize this paper as follows. In Section 2, we derive a large-N_c consistency condition from the pion–nucleon scattering amplitude. This is a standard way to derive the large-N_c commutation relations. In Section 3, we construct the quark representation for the spin and isospin group SU(4), and realize the large-Nc algebra by the method of group contraction. We calculate g_A and $g_A^{(0)}$ in the quark representation and how the Skyrme model and the quark models are (and are not) reproduced by varying a parameter involved in the representation, the number of colours N_c. In Section 4, we consider the two dynamical effects, and see how the problem raised in the previous section is resolved. Furthermore, we present a simple estimate for g_A and $g_A^{(0)}$ in the algebraic framework. The final section is devoted to a brief summary of this contribution. Many of the results discussed in this paper has been published elsewhere [11].

2. Large-N_c Algebra

In this section we derive the large-N_c commutation relations from a consistency condition for large-N_c QCD. Let us consider pion–nucleon scattering. To leading order, two Born terms contribute to the scattering amplitude $\mathcal{M}$ (see Fig. 1):

$$\mathcal{M} \sim N_c \sum_Y \left\{ \frac{\mathcal{V}^\beta_{XY}\mathcal{V}^\alpha_{YZ}}{\omega + M_Z - M_Y} - \frac{\mathcal{V}^\alpha_{XY}\mathcal{V}^\beta_{YZ}}{\omega - M_X + M_Y} \right\} . \tag{2}$$

Here $\sqrt{N_c}\mathcal{V}^\alpha_{XY}$ are the Yukawa vertices with the $\sqrt{N_c}$ dependence factored out, and $M_{X\cdots}$ are the masses of the baryons. Subscripts X, Y, $\cdots$ specify the quantum numbers for the baryons which are in the fundamental representations

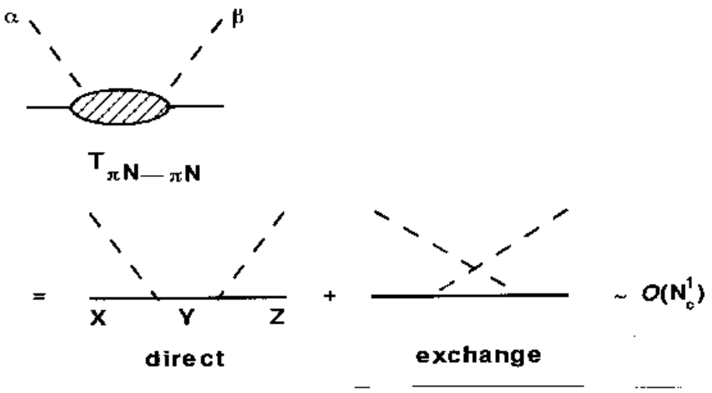


Fig. 1. Born diagrams for pion–nucleon scattering.

of the spin and isospin symmetries, $X \equiv (J(\text{spin}), I(\text{isospin}))$ etc., while α, β stand for the quantum numbers of the pions. The latter label the adjoint representations of the spin and isospin group, since the pion carries unit isospin and spin (P-wave coupling) into the vertices. From the N_c dependence of the two Yukawa couplings, the scattering amplitude superficially grows proportional to N_c.

However, from unitarity or from Witten's large-N_c counting rule [12], the scattering amplitude must be of order $N_c^0 \sim 1$. This is accomplished if the two terms in (2) cancel. Since baryons are degenerate in the limit $N_c \to \infty$, the following relations are sufficient for the cancellation [1]:

$$\sum_Y \left(\mathcal{V}^{\beta}_{XY}\mathcal{V}^{\alpha}_{YZ} - \mathcal{V}^{\alpha}_{XY}\mathcal{V}^{\beta}_{YZ}\right) \xrightarrow{N_c\to\infty} 0\,. \tag{3}$$

Here the left hand side must be suppressed at least to order $1/N_c$. Eqs (3) are regarded as matrix equations for $\mathcal{V}^{\alpha}$ whose components are $\mathcal{V}^{\alpha}_{XY}$.

Combining the spin and isospin group $K = \mathrm{SU}(2) \times \mathrm{SU}(2)$ with the group generated by (3), we obtain the following large-N_c commutation relations:

$$[\mathcal{J}^a, \mathcal{J}^b] = if^{abc}\mathcal{J}^c\,, \quad [\mathcal{J}^a, \mathcal{V}^{\alpha}] = it(a)^{\alpha}_{\beta}\mathcal{V}^{\beta}\,, \quad [\mathcal{V}^{\alpha}, \mathcal{V}^{\beta}] = 0\,. \tag{4}$$

Here $\mathcal{J}^a \in K$ and f^{abc} are the generators and the structure constants for the group K, while $t(a)^{\alpha}_{\beta}$ are the transformation coefficients for $\mathcal{V}^{\alpha}$ under $\mathcal{J}^a$: $t(a)^{\alpha}_{\beta} \equiv \langle\alpha|J^a|\beta\rangle$. Eq. (4) shows that $\mathcal{J}^a$ and $\mathcal{V}^{\alpha}$ form the non-compact group algebra $G = K \times T$, the semi-direct product of K by the Abelian group T generated by $\mathcal{V}^{\alpha}$. This group structure is similar to that of the Lorentz group. Representations of the non-compact group (4) are constructed by the method of induced representation . Here we note the following two results:

(1) The hedgehog state appears as the most symmetric representation of the algebra.

(2) The resulting baryons built upon the hedgehog state form an infinite tower of equal spin (J) and isospin (I), $(I, J) = (\frac{1}{2}, \frac{1}{2}), (\frac{3}{2}, \frac{3}{2}), \cdots \infty$.

3. Contracted SU(4) and the Quark Representation

(3a) Quark Representation

Let us start with counting the number of the generators of $K \times T$, with $K = \mathrm{SU}(p) \times \mathrm{SU}(q)$ and the T a direct product of the adjoint representations of $\mathrm{SU}(p)$ and $\mathrm{SU}(q)$,

$$\#(K \times T) = (p^2 - 1) + (q^2 - 1) + (p^2 - 1)(q^2 - 1) = (pq)^2 - 1 . \tag{5}$$

It is clear that this is the same as the number of the generators of $\mathrm{SU}(pq)$. The commutation relations of $\mathrm{SU}(pq)$ are well known. Let us denote the generators of the subgroup $\mathrm{SU}(p) \times \mathrm{SU}(q)$ by $\mathcal{J}^a$ $(a = 1, 2, ..., p^2 + q^2 - 2)$, and those of the coset $\mathrm{SU}(pq)/\mathrm{SU}(p) \times \mathrm{SU}(q)$ by $\mathcal{W}^\alpha$ $(\alpha = 1, 2, ..., (p^2 - 1)(q^2 - 1))$. The commutation relations are

$$[\mathcal{J}^a, \mathcal{J}^b] = i f^{abc} \mathcal{J}^c,\ [\mathcal{J}^a, \mathcal{W}^\alpha] = i t(a)^\alpha_\beta \mathcal{W}^\beta,\ [\mathcal{W}^\alpha, \mathcal{W}^\beta] = i(g_1^{\alpha\beta\gamma} \mathcal{W}^\gamma + g_2^{\alpha\beta c} \mathcal{J}^c) . \tag{6}$$

Here f, g_1, g_2 are the structure constants. For the case of $p = q = 2$, the constant g_1 vanishes and the last commutators of (6) can be written as

$$[\mathcal{W}^{ia}, \mathcal{W}^{jb}] = \frac{i}{4} \epsilon_{ijk} \delta_{ab} J^k + \frac{i}{4} \epsilon_{abc} \delta_{ij} I^c , \tag{7}$$

where J^i and I^a are the generators of the spin and isospin group.

Consider now N-dimensional symmetric representations of SU(4) and suppose that the matrix elements of $\mathcal{W}$ are of order N. Then the contraction of $\mathrm{SU}(4) \to G = K \times T$ can be performed by replacing $\mathcal{W}^\alpha$ by $\mathcal{V}^\alpha \equiv \mathcal{W}^\alpha / N$. In the limit $N \to \infty$ the algebra (6) of the compact group reduces to the algebra (4) of the non-compact group.

The index N can then be identified with the number of colours in the quark representation. Consider the wave function of a low lying baryon. It is a direct product of orbital, spin, isospin (flavour) and colour parts. Since the colour part is totally antisymmetric, the rest of the wave function must be totally symmetric. For low lying baryons, all quarks are assumed to be in the lowest S state, which gives a symmetric orbital wave function, and thus the spin and isospin part must be totally symmetric. This is the reason that we consider a symmetric state for the spin and isospin group. The symmetric representation of $\mathrm{SU}(4) \supset \mathrm{SU}(2) \times \mathrm{SU}(2)$ is specified by an index N, which is identified with the number of colours N_c.

It is convenient to introduce bosonic operators $\alpha_{\mu\nu}$ $(\mu, \nu = \pm\frac{1}{2})$ for the fundamental representations of SU(4) . For instance, $\alpha_{u\downarrow}$ annihilates a u-quark with down spin. One of symmetric N_c representations is then generated by the hedgehog state

$$|N_c; h_q\rangle = \sqrt{\frac{1}{N_c!}} \left[\frac{1}{\sqrt{2}} (\alpha^\dagger_{u\downarrow} - \alpha^\dagger_{d\uparrow}) \right]^{N_c} |0\rangle , \tag{8}$$

where on the left hand side the subscript q indicates that it is the quark representation. The operator in the square brackets is exactly what is obtained by coupling the spin and isospin to zero grand spin: $\vec{K} = \vec{J} + \vec{I} = 0$. The state (8) breaks spin and isospin symmetries, and all the rotated states

$$|N_c; h_q[A]\rangle \equiv R(A)|N_c; h\rangle \tag{9}$$

$$= \sqrt{\frac{1}{N_c!}} \left[\frac{1}{\sqrt{2}} \left(D^{\frac{1}{2}}_{uu}(A)\alpha^\dagger_{u\downarrow} + D^{\frac{1}{2}}_{du}(A)\alpha^\dagger_{d\downarrow} - D^{\frac{1}{2}}_{dd}(A)\alpha^\dagger_{d\uparrow} - D^{\frac{1}{2}}_{ud}(A)\alpha^\dagger_{u\uparrow} \right) \right]^{N_c} |0\rangle$$

are degenerate. Here $R(A)$ with $A \in \mathrm{SU}(2)$ is a rotational operator acting on the isospin space and $D^{\frac{1}{2}}_{\mu\nu}(A)$ are the SU(2) D-functions of rank $\frac{1}{2}$.

In large-N_c, the theory becomes semiclassical which is verified by looking at the overlap function,

$$\langle N_c; h_q[A'] | N_c; h_q[A]\rangle = [\cos\theta_{AA'^\dagger}]^{N_c} \ , \tag{10}$$

where $\theta_{AA'^\dagger}$ represents symbolically the relative angle between the 'orientations' A and A'. After an appropriate rescaling of the states this overlap goes to a delta function. This sharp peaking is characteristic of a semiclassical limit.

(3b) Nucleon Matrix Elements

The nucleon state $|N\rangle$ can be projected out from the hedgehog state $|h\rangle$ by the method of generator coordinate projection [1, 3]:

$$|N\rangle \propto \int d[A]\, D^{1/2*}_{t,-m}(A)\, R(A)\, |h\rangle\, . \tag{11}$$

Here the third components of spin and isospin equal m and t. The nucleon matrix elements of an observable $\mathcal{O}_N$ can readily be computed by

$$\mathcal{O}_N = \frac{\langle N|\mathcal{O}|N\rangle}{\langle N|N\rangle}\, , \tag{12}$$

where the denominator is needed for normalization.

Let us now calculate the isoscalar (flavour singlet) axial-vector coupling constants $g_A^{(0)}$ and isovector axial-vector coupling constant g_A. These are the nucleon matrix elements of $S_i \equiv \sum_{n=1}^{N_c} \sigma_i(n)$ and $T_{ai} \equiv \sum_{n=1}^{N_c} \tau_a(n)\sigma_i(n)$.

The actual computation is performed conveniently using the Euler angles α, β and γ for rotation. The computational procedure is straightforward and we just give some of the results here:

$$\langle p\downarrow |S_3| p\downarrow\rangle = -N_c \frac{\displaystyle\int d[A]\, C_\beta^2\, S^2\, C^{N_c-1}}{\displaystyle\int d[A]\, C^{N_c+1}} = -\frac{N_c}{N_c} = -1\, , \tag{13}$$

$$\langle p\downarrow|T_{33}|p\downarrow\rangle = -\frac{N_c}{3}\frac{\displaystyle\int d[A](C^{N_c+1}+C^{N_c-1}S^2)}{\displaystyle\int d[A]\; C^{N_c+1}} = -\frac{N_c}{3}\left(1+\frac{2}{N_c}\right). \tag{14}$$

Here we have introduced the notations $C_\beta = \cos\dfrac{\beta}{2}$, $C = \cos\dfrac{\beta}{2}\cos\dfrac{\alpha+\gamma}{2}$, $S = \sin\dfrac{\alpha+\gamma}{2}$, and $d[A] = \sin\beta\, d\beta\, d\alpha\, d\gamma$. The minus signs appear because the matrix elements are for the $p\downarrow$ state. The result (13), $\langle p\downarrow|S_3|p\downarrow\rangle = -1$, which is independent of N_c is trivial, since the nucleon spin here is carried entirely by the intrinsic quark spin. This is nothing but the result of the non-relativistic quark model. In contrast, (14) has an explicit N_c dependence; it becomes $\frac{5}{3}$ when $N_c = 3$ as in the quark model, while it approaches $N_c/3$ in the limit $N_c \to \infty$ as corresponding to the Skyrme model result. This fact is the basis for the general argument that the quark representation produces both the quark and the Skyrme model results [3]. However, it is now apparent that this is not the case for $g_A^{(0)}$.

4. Other Representations

In this section we consider other algebraic realizations for the group SU(4), which are contracted to the large-N_c algebra in the limit $N_c \to \infty$.

(4a) Relativistic Effects

A major relativistic effect is a mixture of the $L = 1$ state through the lower component in the quark wave function:

$$\psi \sim \begin{pmatrix} c_1 \\ c_2\,\vec{\sigma}\cdot\hat{r} \end{pmatrix}. \tag{15}$$

Here the coefficients c_1 and c_2, satisfying the normalization condition $|c_1|^2+|c_2|^2 = 1$, dictates the ratio of the mixture of the upper and lower components* and cannot be determined in the present framework.

Formally such an extension is performed in the algebraic framework by extending the spin group $\mathrm{SU}(2)_S$ to that of the total angular momentum $J = S + L$ and take its diagonal subgroup: $\mathrm{SU}(2)_S \times \mathrm{O}(3)_L \supset \mathrm{SU}(2)_{J=S+L}$, where the generators of the diagonal group is the sum $\vec{J} = \vec{S} + \vec{L}$. The $\mathrm{SU}(2)_J$ group is now combined with the isospin $\mathrm{SU}(2)_I$ group to form the desired $\mathrm{SU}(4) \supset \mathrm{SU}(2)_J \times \mathrm{SU}(2)_I$.

Explicitly, the relativistic hedgehog quarks are written as

$$|N_c; h\rangle \;=\; \begin{pmatrix} c_1\left[[\frac{1}{2},0]^{J=1/2},\frac{1}{2}\right]^{K=0} \\ c_2\left[[\frac{1}{2},1]^{J=1/2},\frac{1}{2}\right]^{K=0} \end{pmatrix}^{N_c} |0\rangle \;\equiv\; \left[\begin{pmatrix} c_1 \\ c_2\vec{\sigma}\cdot\hat{x} \end{pmatrix}\chi^\dagger\right]^{N_c} |0\rangle\,. \tag{16}$$

* Actually, c_1 and c_2 are functions of the radial distance r. In the following discussions, however, such a detailed structure of the wave function is not necessary, and we simply ignore it.

Here the coupling scheme is $[[S,L]^J,\, I]^K$ and $\chi^\dagger \equiv \left[[\frac{1}{2},\, 0]^{J=1/2},\, \frac{1}{2}\right]^{K=0} = \sqrt{\frac{1}{2}}(\alpha^\dagger_{u\downarrow} - \alpha^\dagger_{d\uparrow})$. The last expression of (16) is useful in actual computation. In order to write this expression one should interpret χ as a column vector in spin space. The matrix σ then acts on those spin components, and couples them to the $O(3)$ wave function $\hat{x}$ to form the hedgehog state $[[\frac{1}{2},1]^{\frac{1}{2}},\frac{1}{2}]^0_0$.

The matrix elements for S_3 and T_{33} are computed in a straightforward manner and the results are

$$\langle p\downarrow|S_3|p\downarrow\rangle = -(|c_1|^2 - \tfrac{1}{3}|c_2|^2) \equiv -g_A^{(0)}\,, \tag{17}$$

$$\langle p\downarrow|T_{33}|p\downarrow\rangle = -(|c_1|^2 - \tfrac{1}{3}|c_2|^2)\frac{N_c}{3}\left(1+\frac{2}{N_c}\right) \equiv -g_A\,. \tag{18}$$

We find the same suppression factor $\left(|c_1|^2 - \frac{1}{3}|c_2|^2\right)$ for both the matrix elements. This is understood such that a part of the nucleon spin (with and without isospin factor) is carried by the pion angular momentum ($L=1$) and the fraction of the quark intrinsic spin is suppressed.

In the MIT bag model the suppression factor is $0\cdot 654$ [13]. Accordingly, $g_A^{(0)} = 0\cdot 654$ and $g_A = 5/3 \cdot 0\cdot 654 = 1\cdot 09$. If we wish to reproduce the experimental value of $g_A^{(0)}$ we need a very smaller suppression factor of about $\frac{1}{4}$. This implies a too small $g_A \sim \frac{5}{3}\times\frac{1}{4} \sim 0\cdot 4$. The inclusion of the asymptotic one pion contribution does not help (see Fig. 3 as well as the discussions below). It increases g_A by about 50% [14], but the net value of about $0\cdot 6$ is still too small compared with the experimental value $g_A(\mathrm{exp}) = 1\cdot 25$. Since the relativistic effect is just an overall factor for both g_A and $g_A^{(0)}$, the quark representation with the relativistic effect included cannot produce the quark model and the Skyrme model results by simply varying N_c.

(4b) Pion Effects

We propose that pion effects will be more important for a realistic description of nucleon properties and also for discussing the quark and the Skyrme model results on the same footing. Phenomenologically, the pion is known to be important for such quantities as the neutron charge radius and magnetic moments.

In the large-N_c baryons, the pion cloud together with the quark core form a hedgehog state. The $K=0$ state of the pion is formed by the coupling of its unit isospin and orbital angular momentum. The relevant group for the pion is then $\mathrm{O}(3)_I \times \mathrm{O}(3)_L$, which we once more imbed in an SU(4) group. Therefore, the extended group for the hedgehog quarks and the pions is

$$\mathrm{SU}(4)_q \times [\mathrm{O}(3)_I \times \mathrm{O}(3)_L]_\pi \subset \mathrm{SU}(4)_q \times \mathrm{SU}(4)_\pi\,. \tag{19}$$

Once again, we pick up the diagonal subgroup, $\mathrm{SU}(4)_{q+\pi} \subset \mathrm{SU}(4)_q \times \mathrm{SU}(4)_\pi$.

We consider an ansatz of the direct product of the N_c quarks and N_π pions. The $\mathrm{SU}_\pi(4)$ algebra can be realized in terms of the same type of bosonic operators as for the quark algebra, which we denote as $\beta_{\mu\nu}$. The pion hedgehog can now be expressed as $\left[\sqrt{\frac{1}{2}}(\beta^{\dagger}_{u\downarrow} - \beta^{\dagger}_{d\uparrow})\right]^2$. This has the correct symmetries expected for a $\bar{q}q$ state, and has the spin and isospin structure $([\frac{1}{2}, \frac{1}{2}]^0_0)^2$. Note that this construction includes the σ-like excitations as well as those of the pion. However, their combination is chiral symmetric just as $U \sim \sigma + i\vec{\tau}\cdot\vec{\pi}$ of chiral models.

Now the hedgehog state takes the direct product of the quark and pion hedgehog:

$$|N_c; h_{q+\pi}\rangle = \sqrt{\frac{1}{N_c!}}\left[\frac{1}{\sqrt{2}}(\alpha^{\dagger}_{u\downarrow} - \alpha^{\dagger}_{d\uparrow})\right]^{N_c} |0\rangle$$

$$\times \sqrt{\frac{1}{(2N_\pi)!}}\left[\frac{1}{\sqrt{2}}(\beta^{\dagger}_{u\downarrow} - \beta^{\dagger}_{d\uparrow})\right]^{2N_\pi} |0\rangle \,. \tag{20}$$

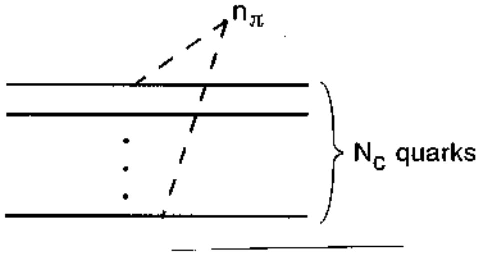


Fig. 2. Counting of the pion number in the nucleon. The pion–quark Yukawa vertex is of order $N_c^{-1/2}$, while the combination factor of picking up two quark lines out of N_c is $N_c(N_c-1)$. This makes the number of pions of order N_c.

The meaning of the number of the pions is not very clear, and perhaps it would be difficult to give it a precise dynamical meaning. But it is related, as shown later shortly, to how the nucleon spin is partitioned between quarks and pions. In a large-N_c baryon of an N_c quark bound state, the number of pions in the baryon is expected to be proportional to N_c, $N_\pi = cN_c$. This is understood from a diagrammatic counting as shown in Fig. 2. The coefficient c, however, cannot be determined in the present algebraic framework.

We calculate the nucleon matrix elements for S_i and T_{ia}. Note that those operators act only on the N_c quarks. The results are

$$\langle p\downarrow|S_3|p\downarrow\rangle = -\frac{N_c}{N_c + 2N_\pi} \equiv -g_A^{(0)} \,, \tag{21}$$

$$\langle p\downarrow|T_{33}|p\downarrow\rangle = -\frac{N_c}{3}\left(1 + \frac{2}{N_c + 2N_\pi}\right) \equiv -g_A \,. \tag{22}$$

These results, though simple, have interesting implications. The nucleon spin $g_A^{(0)}$ becomes less than unity for a finite number of pions $N_\pi \neq 0$, where a part of the nucleon spin is carried by the angular momentum of pions. When $N_\pi = 0$, Eq. (21) reduces to the quark model result, where the entire nucleon spin is carried by the quark spin. The Skyrme model result of vanishing $g_A^{(0)}$ [15] is now obtained when $N_\pi \to \infty$, where the nucleon spin is entirely carried by the pion cloud [16]. The same thing holds also for the second term of g_A in (22). In particular, the Skyrme model limit when $N_\pi \to \infty$, rather than when $N_c \to \infty$, is interesting. This result may be understood by interpreting the Skyrme soliton as a coherent superposition of infinitely many pions in the classical limit.

(4c) Simple Estimates

Let us make a rough estimate for $g_A^{(0)}$ and g_A including both the relativistic and pion effects. The importance of the latter two effects have been also discussed in Ref. [17]. The result in the algebraic method here is simply multiplying the same reduction factor due to the relativistic effects to (21) and (22). A reasonable estimate for that would be $\sim 0{\cdot}7$. For $N_c = 3$, g_A then becomes

$$g_A \sim 0{\cdot}7\left(1+\frac{2}{3+N_\pi}\right). \tag{23}$$

The one pion pole contribution as depicted in Fig. 3 must be added to this result. This is entirely related to the chiral symmetry aspect of the theory and cannot be included in the present algebraic framework. In the chiral limit the pole contribution is calculated exactly: it is precisely 50% of the quark contribution [14]. So we get the total value of g_A to be

$$g_A \sim 1{\cdot}5 \times 0{\cdot}7\left(1+\frac{2}{3+N_\pi}\right) \sim \left(1+\frac{2}{3+2N_\pi}\right). \tag{24}$$

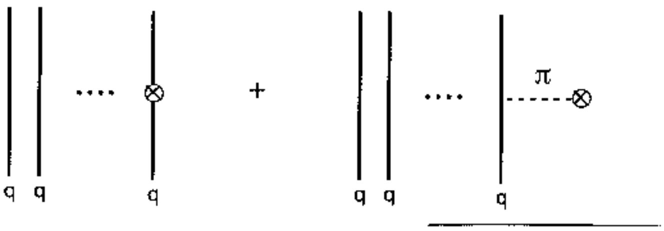


Fig. 3. Two components of the axial vector coupling constant g_A: one is the quark (or short range) part and the other is the asymptotic one pion contribution. The second term, which is precisely 50% of the first term in the chiral limit, cannot be included in the present algebraic method.

The experimental value of $g_A \sim 1{\cdot}3$ is then reproduced when $N_\pi \sim 2$. Note that the $1/N_c$ correction term in (24) is about 30%, which is in good agreement with the numbers in various chiral models . Using the same parameters, the nucleon spin $g_A^{(0)}$ turns out to be

$$g_A^{(0)} \sim 0{\cdot}7\frac{3}{3+2N_\pi} \sim 0{\cdot}3\,.$$

It is remarkable that such a simple algebraic method can be used to describe both g_A and $g_A^{(0)}$ simultaneously in good agreement with experiments.

5. Summary

In this contribution we have investigated algebraic models that contain a spin and isospin subgroup SU(4) whose contraction reduces to the large-N_c algebra for QCD. We have explicitly constructed a realization which interpolates the Skyrmion and the quark model results for $g_A^{(0)}$ and g_A by combining a possible pionic effect with the familiar quark representation. We have calculated axial vector coupling constants and shown that they are nicely reproduced in the present method. Those two quantities are in fact matrix elements of the generators of the SU(4) algebra, which is the primary reason that we could make a reliable prediction without referring to detailed dynamical information. It would be interesting to extend the algebraic method further to other quantities such as mass and magnetic moment.

Acknowledgments

A.H. acknowledges the Adelaide Institute of Theoretical Physics for its hospitality and for partial support.

References

[1] J.-L. Gervais and B. Sakita, Phys. Rev. Lett. **52** (1984) 87; Phys. Rev. **D30** (1984) 1795.
[2] K. Bardakci, Nucl. Phys. **B243** (1984) 197.
[3] A.V. Manohar, Nucl. Phys. **B248** (1984) 19.
[4] R. Dashen and A.V. Manohar, Phys. Lett. **B315** (1993) 425; *ibid.* 438.
[5] E. Jenkins, Phys. Lett. **B315** (1993) 431; *ibid.* 441; *ibid.* 447.
[6] R. Dashen, E. Jenkins and A.V. Manohar, Phys. Rev. **D49** (1994) 4713; *ibid.* **D51** (1995) 3697.
[7] C. Carone, H. Georgi and S. Osofsky, Phys. Lett. **B322** (1994) 227.
[8] M.A. Luty and J. March-Russell, Nucl. Phys. B426 (1994) 71; M.A. Luty, Phys. Rev. **D51** (1995) 2322.
[9] R.D. Amado, R. Bijker and M. Oka, Phys. Rev. Lett. **58** (1987) 654.
[10] M. Oka, R. Bijker, A. Bulgac and R.D. Amado, Phys. Rev. **C36** (1987) 1727.
[11] A. Hosaka and N.R. Walet, Phys. Lett. **B358** (1995) 184.
[12] E. Witten, Nucl. Phys. **B160** (1979) 57.
[13] A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn, Phys. Rev. **D10** (1974) 2599; T. DeGrand, R.L. Jaffe, K. Johnson, J. Kiskis, Phys. Rev. **D12** (1975) 2060.
[14] R.L. Jaffe, *In* 'Pointlike Structures Inside and Outside Hadrons' (Ed. A. Zichichi), p. 99 (Plenum, New York, 1982).
[15] S. Brodsky, J. Ellis and M. Karliner, Phys. Lett. **B206** (1988) 309.
[16] M. Wakamatsu, Phys. Lett **B232** (1989) 251; **B234** (1990) 223.
[17] R. Johnson *et al.*, Phys. Rev. **D42** (1990) 2998.
[18] A. Hosaka and H. Toki, Phys. Lett. **B322** (1993) 1; **B343** (1994) 1; NCT/RCNP preprint (1995), submitted to Phys. Rep.
[19] M. Wakamatsu and T. Watabe, Phys. Lett. **B312** (1993) 184.
[20] A. Blotz, M. Praszalowicz and K. Goeke, Phys. Lett. **B317** (1993) 195.