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The electrical potential arising from a point source in an arbitrarily anisotropic half-space with regolith cover

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ABSTRACT

Solutions for the electrical potential and current density due to a point source in an arbitrarily anisotropic half-space with a regolith cover have been obtained, using an image source method. The images of the point source include a point and a line arising from the surface boundary conditions. The source density of the line image obeys an exponential law. The line image position may be described by linear parametric equations. Within this approximation the solutions provide a relatively complete description of the potential in the arbitrarily anisotropic half-space covered by a thin surface layer.

Keywords: Anisotropy, regolith cover, potential, analytical solution, image source

INTRODUCTION

Most prospective areas, especially for mineral, in Australia are covered by superficial cover, called the regolith, which can screen or distort the geophysical signals from deeper mineral deposits. Hence knowledge of the behaviour of the electric potential in an anisotropic half-space with a thin cover is of considerable importance in the field of mineral exploration. Figure 1 is a schematic diagram the geometry of the present study.

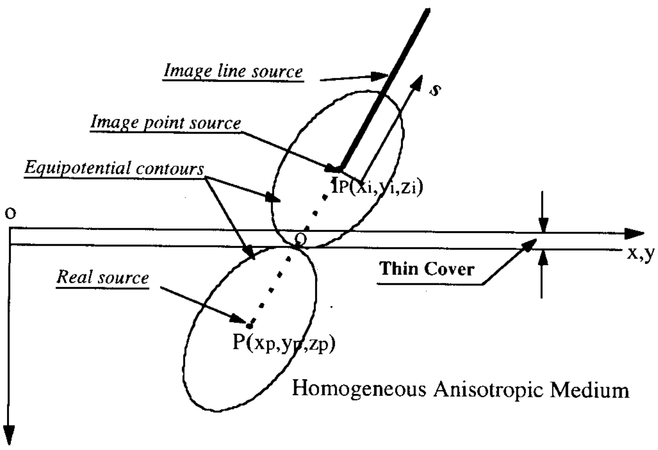

Figure 1. Point source images in an anisotropic half-space with a thin cover.

A previous solution for this problem has been developed by Lindell et al. (1993) for a point source in an anisotropic half-space with an infinitesimally thin boundary layer using the image method. This paper is an improvement on that method. A layer of thickness Δh is considered to cover the surface of the half-space.

THEORY AND METHOD OF SOLUTION

Mathematical definition of the problem

The problem can be formulated mathematically by applying boundary conditions to an appropriate form of the solution of the governing equation. The governing equation is:

$$\nabla \cdot \sigma_{i,j} \cdot \nabla v = I_0 \delta(r - r_0), \qquad (1)$$

where $\sigma_{i,j}$ is the conductivity tensor which is symmetric and positive definite and $I_0\delta(r-r_0)$ is a point source function.

If the Earth surface is covered by a thin layer, equation (1) is subjected to the following boundary conditions,

$$\vec{E_t} = c\sigma_t \cdot \vec{J_s}, \qquad (2)$$

where $\vec{E_t}$ is the transverse part of the electric field strength in the surface layer, σ_t is the transverse part of the conductivity tensor, and $\vec{J_s}$ is the current density on the air-Earth surface. The conductivity contrast parameter is $c = \sigma_t / \sigma_t^c$, where σ_t^c is the transverse part of the conductivity tensor of the cover. The second boundary condition requires the potential to reduce to zero at points infinitely far from the source.

The image source functions

A point image current source alone cannot satisfy the boundary conditions. Lindell et al. (1993) gave a theoretical model that suggests that the complete image is formed by a combination of a point source and an exponential line source. In this paper the image sources is presented as:
(1) point image source

$$s_i = -I_0 \delta(R), \qquad (3)$$

where $R(x_i,\ y_i,\ z_i)$ is the image point coordinate; together with (2) line image source

$$s_l = 2c\sqrt{r_x^2 + r_y^2 + 1}\, e^{-cs\sqrt{r_x^2 + r_y^2 + 1}}, \qquad (4)$$

the source location being along the straight line passing through the original source point P and the image point I^P. The s value starts from $-z_i$. r_x and r_y are the anisotropy divergences given by Li and Uren (1996) as:

$$r_x = \frac{\varsigma_{2,2}\varsigma_{1,3} - \varsigma_{1,2}\varsigma_{2,3}}{\varsigma_{2,2}\varsigma_{1,1} - \varsigma_{1,2}^2}, \qquad (5)$$

$$r_y = \frac{\varsigma_{1,1}\varsigma_{2,3} - \varsigma_{1,2}\varsigma_{1,3}}{\varsigma_{2,2}\varsigma_{1,1} - \varsigma_{1,2}^2},$$

where $\varsigma_{i,j} = \sigma_{i,j}^{-1}$.

Solution of the potential due to the sources

The solution of the governing equation (1) is:

$$v=\frac{I}{4\pi}\left[\frac{1}{|\eta|}-\frac{1}{|\eta|}+\int_{-z'}^{\infty}\frac{2e^{-cs\sqrt{r_x^2+r_y^2+1}}\sqrt{r_x^2+r_y^2+1}ds}{|\eta(s)|}\right], \tag{6}$$

where $I=I_0(\det[\varsigma_{i,j}])^{1/2}$,

$|\eta|=[\varsigma_{1,1}X^2+\varsigma_{2,2}Y^2+\varsigma_{3,3}Z^2+2\varsigma_{1,2}XY+2\varsigma_{1,3}XZ+2\varsigma_{2,3}YX]^{1/2}$,

$|\eta'|=[\varsigma_{1,1}X_r^2+\varsigma_{2,2}Y_r^2+\varsigma_{3,3}Z_r^2+2\varsigma_{1,2}X_rY_r+2\varsigma_{1,3}X_rZ_r+2\varsigma_{2,3}Y_rX_r]^{1/2}$,

$|\eta(s)|=[\varsigma_{1,1}X(s)^2+\varsigma_{2,2}Y(s)^2+\varsigma_{3,3}Z(s)^2+2\varsigma_{1,2}X(s)Y(s)+$
$2\varsigma_{1,3}X(s)Z(s)+2\varsigma_{2,3}Y(s)X(s)]^{1/2}$,

$X=x-x^p$, $Y=y-y^p$, $Z=z-z^p_r$ $X_r=x-x^r$, $Y_r=y-y^r$, $Z_r=z-z^r$
$X(s)=x-r_xs$, $y(s)=y-r_ys$ and $Z(s)=z-s$.

NUMERICAL EXAMPLE

Consider an arbitrarily anisotropic half-space with the following conductivity tensor whose major axis dips at 68.1° and azimuth angle is 15.8° from the x-axis.

$$\sigma_{i,j}=\begin{pmatrix}0.08 & 0.005 & -0.08\\ 0.005 & 0.2 & -0.1\\ -0.08 & -0.1 & 0.3\end{pmatrix} \tag{7}$$

Figure 2 (a, b, c) shows the equipotential on a vertical plane at y=5.0 m with the source at (0.0, 0.0, 5.0). The surface layer is 0.1 m thick, and Figure 2 (a, b, c) shows conductivity contrasts of 1.0 (no cover), 0.3 and 0.4 respectively. Those in Figure 2(d), 2(e) and 2(f) are the equipotentials on the Earth surface, with the same source location and the conductivity contrasts as in Figures 2(a), 2(b) and 2(c). It is observed that with increasing values of c, the influence of the surface cover on the equipotential contours is stronger.

CONCLUSIONS

A solution for the electric potential in an arbitrarily anisotropic and homogeneous earth with a regolith cover has been found. The source function was introduced in this general solution by using a point and line image based on the theory of Lindell et al. (1993) and the work of Li and Uren (1996). The technique is computationally efficient, compared to numerical methods such as Fourier transforms and finite difference methods. Other numerical experiments show hat when the source is located about 50 m the surface and the cover thickness is 1 m, the results are still correct. Using this basic theory, the performance of any common electrical prospecting geometry may be simulated for the model presented in this paper.

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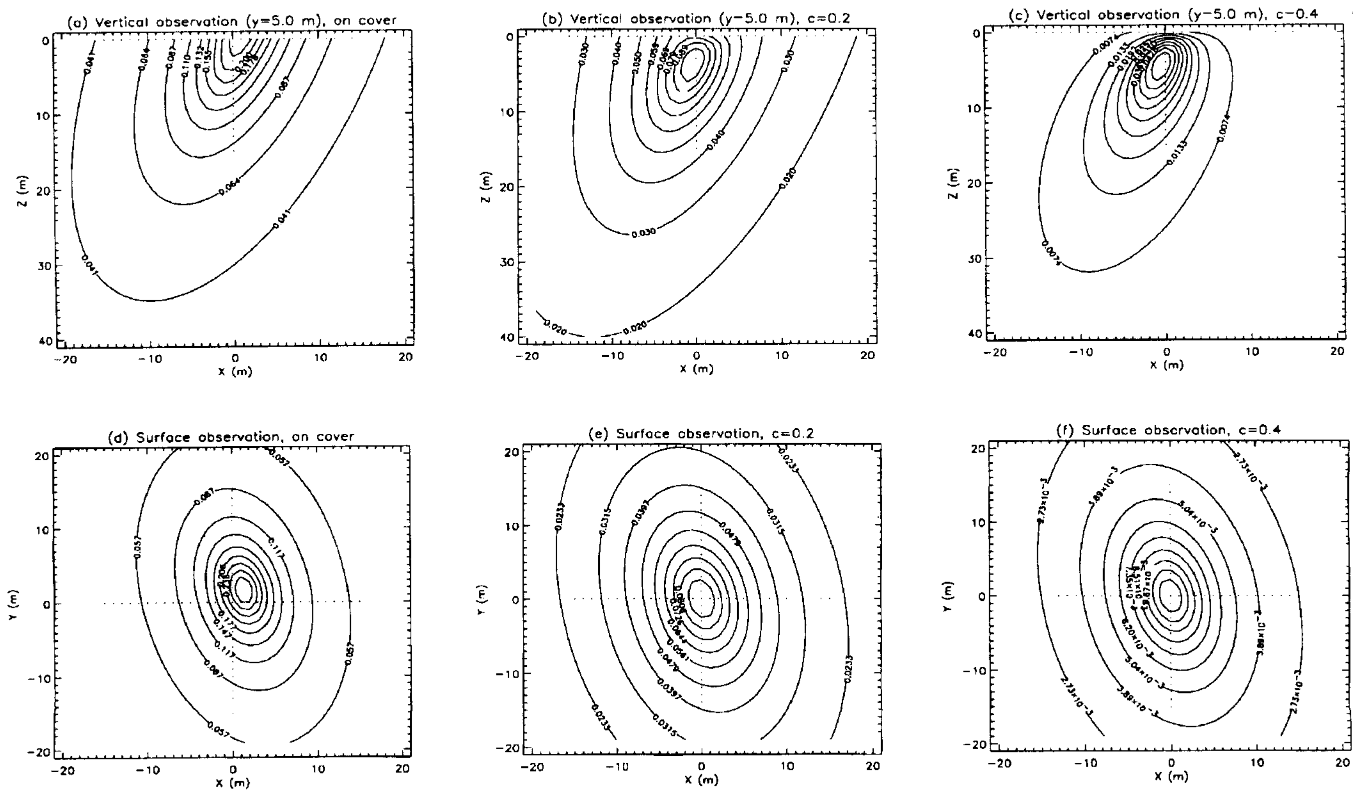

Figure 2. Equipotential contours arising from a D.C. source located at (0.0, 0.0, 5.0) with conductivity contrast 1.0 (no cover), 0.2 and 0.4. (a), (b) and (c) are the observed equipotential contours on the vertical plane y=5.0 m. (d), (e) and (f) are the equipotential contours observed on the Earth's surface.