*Exploration Geophysics* (1989) **20** 487–489

# Transient Electromagnetic Fields for a Polarized Conductive Sheet

**James R. Wait**
*2210 East Waverly*
*Tucson AZ*
*85719 USA*

## Abstract

We formulate the problem of a transient magnetic dipole located over a thin conductive sheet which may exhibit frequency dispersion or I.P. (induced polarization). A first order analysis is carried through for the case when the source current is a step function. It is shown that the I.P. slow tail can be of opposite polarity to the principal electromagnetic response.

## Introduction

In geophysical exploration and non-destructive testing, the thin sheet conductor is a useful idealization of the target. In such cases, the conductivity-thickness product, $\sigma d$, or conductance is a key parameter. When the sheet, located in free space, is excited by an external pulsed magnetic dipole, it has been shown that the image of the source is a like dipole but its distance from the sheet recedes in time (Wait, 1956). Such a concept was originally put forth by Maxwell (1891) a century ago. The normalized time parameter, for such a problem, is $t/(\mu_0 \sigma d l)$ where $\mu_0 = 4\pi \times 10^{-7}$ is the permeability of the surrounding free space, and l is some scale length such as the distance of the source to the sheet.

An interesting extension of the thin sheet induction problem is to allow the conductivity of the sheet to be a function of frequency which we designate as $\sigma(jw)$ for a time factor exp(jwt). We then enquire if the image concept is still applicable. And, if so, what modifications of the time domain formulation are needed? For purposes of illustration, we choose a simple Debye type model for the frequency dispersion of the sheet conductivity. A closely related problem was discussed recently in this journal by Smith and West (1988). We wish to complement their development with an approximate analysis; and it is hoped that the new insight provided is not entirely trivial.

## Basic Transform Representation

The formal solution of the quasi-static time harmonic problem is well known (e.g. Wait 1951, Grant and West 1965) so we may summarize the solution very briefly here. To cast the needed result into Laplace transform notation, we merely replace jw by s, the transform variable. Also, we define a normalized sheet admittance y(s) by the relation

$$y(s) = \sigma(s)\, d\mu_0/2 \tag{1}$$

The conductive sheet, itself, is to be located in the plane z = 0 of a cylindrical coordinate system (r, $\phi$, z). The source, located at z = h, is a small loop of area dA carrying a current I(t) which is zero for t < 0. The corresponding transform is obtained from

$$I(s) = \int_0^\infty I(t)\, e^{-st}\, dt = L\, I(s) \tag{2}$$

where L is the Laplace transform operator.

Now, within our quasi-static assumption, the magnetic field outside the sheet can be obtained from a time-varying magnetic potential U(t) according to

$$\vec{H}(t) = -\operatorname{grad} U(t) \tag{3}$$

where

$$U(t) = L^{-1}\, U(s) \tag{4}$$

where $L^{-1}$ is the inverse Laplace transform operator. For z > 0, we may write

$$U(s) = -\frac{I(s)\, dA}{4\pi}\left[\frac{\partial}{\partial z}\frac{1}{R} - \frac{\partial}{\partial z}\int_0^\infty s\, f(s)\, e^{-k(z+h)}\, J_0(kr)\, dk\right] \tag{5}$$

where

$$f(s) = [\, s + (k/y(s))\,]^{-1} \tag{6}$$

and

$$R = [\, r^2 + (z-h)^2\,]^{\frac{1}{2}} \tag{7}$$

## Explicit Inversion of Transform

To specialize further, we will now adopt a turn-on or step function current excitation. Thus

$$I(t) = I_0\, u(t) \tag{8}$$

where u(t) = 0 for t < 0 and u(t) = 1 for t > 0. Then we deduce that the time dependent potential is

$$U(t) = -\frac{I_0 dA}{4\pi}\left[\frac{\partial}{\partial z}\frac{1}{R} - \frac{\partial}{\partial z}\int_0^\infty [\, L^{-1} f(k,s)\,]\, e^{-k(z+h)} J_0(kr)\, dk\,\right] u(t) \tag{9}$$

Now we restrict attention to weakly dispersive conductivity such that the fractional change of $\sigma(jw)$, over all significant frequencies, is small compared with one. Thus we may write

$$f(k,s) \cong f_\infty(k,s) + [y(s) - y_\infty] f_\infty'(k,s) \quad (10)$$

Here $y_\infty = \lim_{s \to j\infty} y(s)$ is the high frequency limit of the normalized admittance. Furthermore, in our example,

$$f_\infty(k,s) = 1/[s + (k/y_\infty)] \quad (11)$$

and

$$f_\infty'(k,s) = \frac{\partial f(k,s)}{\partial y(s)}\Big|_{y(s) \to y_\infty} = \frac{\partial f_\infty(k,s)}{\partial y_\infty} \quad (12)$$

Thus, it follows that

$$f_\infty'(k,s) = (k/y_\infty^2)[s + (k/y_\infty)]^{-2} \quad (13)$$

Equation (9) can now be decomposed as follows

$$U(t) = U^{EM}(t) + U^{IP}(t) \quad (14)$$

The first part is

$$U^{EM}(t) = -\frac{I_o dA}{4\pi}\left[\frac{\partial}{\partial z}\left(\frac{1}{R} - \frac{1}{R'(t)}\right)\right]u(t) \quad (15)$$

where

$$1/R'(t) = \int_0^\infty L^{-1}[s + (k/y_\infty]^{-1} e^{-k(z+h)} J_o(kr)\, dk \quad (16)$$

$$= \int_0^\infty e^{-(k/y_\infty)t} e^{-k(z+h)} J_o(kr)\, dk \; u(t) \quad (17)$$

$$= 1/[r^2 + (z+h+(t/y_\infty))^2]^{\frac{1}{2}}\; u(t) \quad (18)$$

Carrying out the differentiation, we obtain

$$U^{EM}(t) = \frac{I_o dA}{4\pi}[(z+h)/[(r^2 + (z+h)^2]^{3/2} \quad (19)$$

$$- (z+h+t/y_\infty)/[r^2 + (r^2 + (z+h+t/y_\infty)^2]^{3/2}]$$

Clearly $U^{EM}(t)$ is the electromagnetic response computed as if the conductivity of the sheet was $\sigma(d\ \infty)$ or $\sigma_\infty$ for all frequencies. Then we see that $R''(t)$ is the time dependent distance from the image at $z = -(h + t/y_\infty)$ to the observer at (r, z). Of course, the contribution from the image vanishes as t becomes sufficiently large.

Now the second part of (14) is given by

$$U^{IP}(t) \cong -\frac{I_o dA}{4\pi} L^{-1}[y(s) - y_\infty]\int_0^\infty \frac{k^2}{[s y_\infty + k]^2} e^{-k(z+h)} J_o(kr) dk \quad (20)$$

We identify this as the I.P. response although it also has an electromagnetic contribution if no further approximations are made.

## The Debye Dispersion Model

To proceed further, we need to specify a dispersion model. We adopt the simple Debye form corresponding to a single relaxation time g. Thus

$$\sigma(jw) - \sigma(j\infty) = -m\sigma(j\infty)/[1 + jwg] \quad (21)$$

or equivalently

$$y(s) - y_\infty = -m y_\infty/[1 + sg] \quad (22)$$

Here the dimensionless factor m can be called the chargeability. In essence we have already assumed m is small compared with one being the case for weak polarization.

The relevant transform in (20) to be inverted is now dealt with as follows:

$$L^{-1}(y(s) - y_\infty)k^2/[sy_\infty + k]^2$$

$$= -[mk^2/(y_\infty g)]L^{-1}[(s + 1/g)(s + k/y_\infty)]^{-1} \quad (23)$$

$$= -(mk^2/g y_\infty)$$

$$\frac{[e^{-t/g} + [(1/t - k/y_\infty)t - 1]e^{-kt/y_\infty}]}{[1/g - k/y_\infty]^2} \quad (24)$$

$$\cong -(m y_\infty/g)\exp(-t/g) \quad (25)$$

where the latter approximate form (25) is valid if $g >> y_\infty/k$ for all significant values of k. Here g can be dubbed the I.P. time constant while $y_\infty$ is the E.M. time constant bearing in mind that 1/k is a characteristic scale length. Then, using (20) we see that

$$U^{IP}(t) \cong \frac{I_o dA}{4\pi}(m y_\infty/g)(1/R)\exp(-t/g)\; u(t) \quad (26)$$

where we have used the indentity

$$1/R = \int_0^\infty e^{-k(z+h)} J_o(kr)\, dk \quad (27)$$

where R is given by (7) being the distance from the geometrical image point, at $z = -h$, to the observer at (r, z).

## Concluding Remarks

It is remarkable that the I.P. slow tail, characterized by the time constant g, appears to be equivalent to a time varying magnetic pole at the image point $z = -h$ but it decays with time exponentially. Also it is proportional to the conductivity of the sheet and varies inversely with the I.P. time constant. An alternative interpretation of the 1/R dependence for the potential is to view the pole contribution as a semi-infinite magnetic current extending downwards from $z = -h$ to $-\infty$. But, of course, we should bear in mind that that we have made a number of physical approximations to simplify the analysis. Essentially we have assumed that the dispersion or polarizably is weak (e.g. $m < 0.2$) and that the I.P. time constant is somewhat greater than the E.M. time constant. It would not be difficult to relax such assumptions, as Smith and West (1988) have done in a closely related problem, but then numerical effort is required and some insight is lost.

## References

Grant, F. S., and West, G. F. (1965). 'Interpretation Theory in Applied Geophysics', (McGraw Hill)

Maxwell, J. C. (1891). 'A Treatise on Electricity and Magnetism' (Clarendon Press)

Smith, R. S., and West, G. F. (1988). An explanation of abnormal TEM responses — coincident negatives, and the loop effect'. *Exploration Geophysics*, **19**, 435–436.

Wait, J. R. (1951). 'The magnetic dipole over the horizontally stratified earth'. *Can. Jour. Phys.* **29**, 577–592.

Wait, J. R. (1956). 'Shielding of a transient magnetic field by a conductive sheet'. *Can. Jour. Phys.*, **34**, 890–893.

*(Paper received: 19/9/89)*