Interactive Broadband Constrained Inversion

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Abstract

Conventional algorithms for the inversion of seismic data generally produce inverted traces which are limited in bandwidth to that of the input seismic data. This necessitates that a low-frequency model be added to the inverted traces in order to produce full bandwidth seismic logs. A problem with this method is that there is usually a gap in frequency between the low end of the seismic and the high end of the model spectra. A further problem with conventional inversion methods is that it is difficult to incorporate well or geologic information into the inversion process.

To overcome these problems a new broadband constrained inversion method which combines seismic, well, and geologic information is used. This method simultaneously satisfies constraints imposed by the well and geologic information, whilst inverting the seismic data. The output is an optimized broadband acoustic impedance model.

In order to combine the diverse data types of seismic amplitudes, well logs, geologic models, and horizon interpretations, an interactive workstation is used. The interactive environment is ideal for this work as it provides the flexibility required to manipulate and display both the varying incoming data types and the output data model.

Key words: inversion, constrained inversion, interactive

Introduction

Seismic logs based on the inversion of seismic data are routine products today. Improvements in data acquisition and processing technology provide seismic information that can be directly correlated to lithology. Traditional methods of inversion, whether a trace integration or modelling technique, suffer from being restricted to the bandwidth of the input seismic data. Because seismic data rarely has any frequency components below 5 to 8 Hz, a low-frequency velocity model must be added to the high-frequency inverted seismic traces in order to produce a full-bandwidth seismic log. This low-frequency model is generally derived from either seismic velocities or well velocities or a combination of both and will probably contain information only up to 3 or 4 Hz. This means that there is generally a frequency gap in the spectrum between the low-frequency model, input by the user, and the high-frequency information contained in the seismic data. Recent attempts to address this problem using a non-linear transfer function to extend the low-frequency component of the seismic data into this spectral gap (Oldenburg *et al*, 1983) have had limited success. Additionally, the inverted seismic logs will be bandlimited at the high frequency end of the spectrum to values generally below 100 Hz. This limits the resolution obtainable from conventional inversion profiles. Another problem is the difficulty in incorporating existing well information or geologic models into the inversion process. In particular, the input from well data is usually restricted to its contributions to the low-frequency model, no high-frequency well information is directly used in obtaining the inverted seismic traces. This paper presents a broadband constrained inversion (BCI) method which combines seismic, geologic and well information to obtain an optimized broadband acoustic impedance model. This model simultaneously satisifies the constraints imposed by geologic and well information, whilst inverting the seismic data amplitudes.

The process of inverting data subject to a priori constraints has been previously reported (Jackson, 1979). However, the practical aspects of bringing together the many data types required has limited its applications. Some of the information is quantitative, like the recorded seismic and well data, while some of it is interpretative, such as unit boundaries, fault locations and sequence or facies changes. Furthermore, certain types of information may be finely resolved vertically but irregularly distributed in space (at well locations), in contrast to the uniformly distributed but grossly sampled (in time or depth) seismic data and its horizon interpretation. Interactive workstations provide the necessary flexibility required to manipulate this diversity of data (Figure 1).

Process Description

A flow chart of the broadband constrained inversion process is shown in Figure 2. An initial model is built which combines parameter estimates (e.g. impedance) from well data and interpretative elements such as horizon distributions, geologic

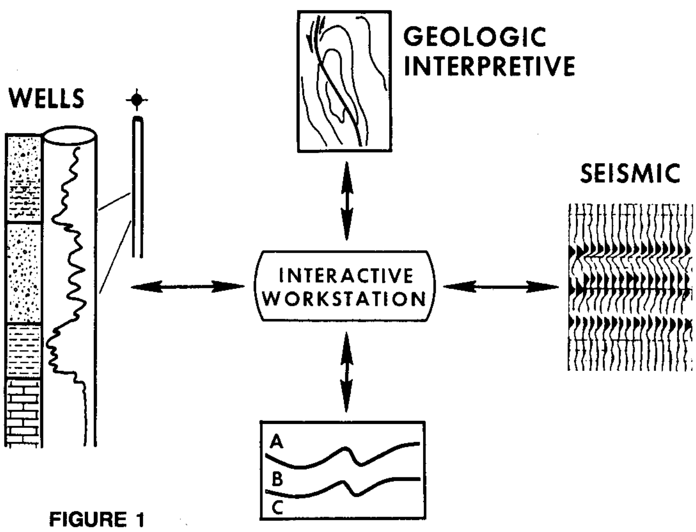

FIGURE 1
Data integration concept utilizing an interactive workstation to combine geologic, well, horizon and seismic data.

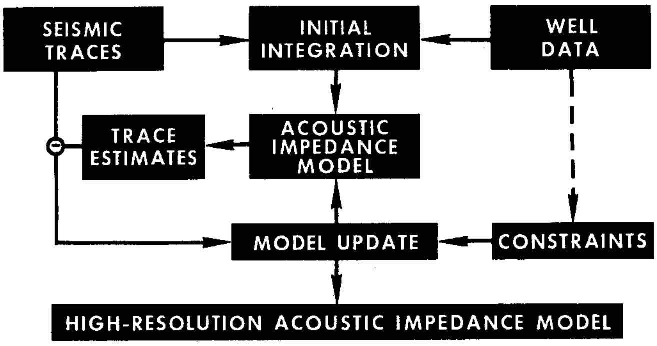

FIGURE 2
Block diagram of the broadband constrained inversion system showing the iterative procedure used.

information, and fault locations. In addition a sphere of influence about each well location is defined. The initial model may be defined for a seismic line, or a well-tie traverse in a 3D data set, or indeed an entire data volume. Traces are then estimated from the initial model and compared with the actual seismic data to yield residual error traces. This error is then used to update the model parameters through a rigorous inversion procedure which meets the constraints imposed by the well information and the geologic/seismic interpretation, while at the same time fitting the seismic data. The updated model may then be used as the starting point for a new inversion loop. Several iterations may be required to converge to a final optimized model.

Each stage of this inversion process will now be described in more detail.

Model Building

The first stage in building the intial model is to link the well data to the seismic times at each well location. Generally, this is done by comparing the times of several marker horizons on the seismic data to the times on synthetic seismograms derived for each well. This information is then used to calculate depth-to-time functions. The acoustic impedance values at each well are then mapped into the time domain using these functions. Next, the seismic data is interpreted to obtain a geologic representation of the subsurface. The vertical component impedance data is then intersected with the horizontal component interpretative data at each well and spatially interpolated using the geologic horizons as guides. The resultant estimated impedance model is then used as the intial model to start the inversion process.

Model Parameterization

The model used by the inversion process is parameterized by the two variables: amplitude and delay time. The delay time is non-integer, and may be made as precise as desired. The reflectivity model is given by:

(1) $r(t) = \Sigma a(i)\delta[t - \tau(i)]$; $i = 1, \ldots, N$

where r (t) is the reflectivity model with N reflectors,
a (i) is the amplitude of the i'th reflector.
τ (i) is the delay time of the i'th reflector,
δ is the Dirac delta function,
t is the time sample index.

The seismic trace is then represented by

(2) $s(t) = r(t) * w(t) + n(t)$; $t = 1, \ldots, M$

where s(t) is the seismic trace with M time samples,
r(t) is the reflectivity model with N reflectors,
w(t) is the bandlimited seismic wavelet,
n(t) is the bandlimited additive noise.

It is assumed that w(t) is known (generally a bandlimited zero-phase wavelet), and that the spetrum of n(t) is also known (generally white).

Substituting equation (1) into equation (2) gives

(3) $s(t) = \Sigma a(i)w[t - \tau(i)] + n(t)$; $i = 1, \ldots, N$

for each time sample index t to M. This equation describes a mapping from the 2N-dimensional parameter space (amplitude and delay time) to the M-dimensional data space of the seismic trace.

Using this parameterization, the initial model is represented by discrete individual events, each with a reflectivity (amplitude) and a time delay, at each seismic trace location. This representation is ideally suited to the horizontal orientation of horizon and geologic interpretations.

Linearization

Each seismic trace is thus a composite of wavelets; one for each event, scaled by the event amplitude and positioned at the event time. However, since this formulation is non-linear in t (time) it must be linearized. This is done by taking the first two terms of its Taylor's Series using the initial model as the point about which the series is expanded (van Riel and Berkhout, 1985). The linearized seismic trace then has the matrix formulation:

(4) $s = W_0 a_0 + G(\tau - \tau_0) + n$

where s is seismic trace vector,
W_0 is the MxN "wavelet convolution" matrix for the initial model,
a_0 is the amplitude vector for the initial model,
τ is the delay times vector of the seismic trace,
τ_0 is the delay times vector for the initial model,
n is the additive noise vector,
G is the Jacobian matrix, defined by

(5) $\Delta s = G\Delta\tau + n$

evaluated at the initial model parameters a_0 and τ_0

where $\Delta s = s - W_0 a_0$ and $\Delta\tau = \tau - \tau_0$ Thus, $\Delta\tau$ is the timing error of each event and the seismic trace is linearized about the initial model via the Jacobian G the partial derivatives of δ with respect to τ.

Stochastic Inversion

The model and the seismic data are combined using a constrained linear inversion procedure which optimizes the output model. A stochastic algorithm is used for the inversion. The initial model is perturbed by the algorithm to produce an updated model estimate, m. This estimate is given by:

$$(6)\quad m = m_0 + [G^TG + C_NC_M^{-1}]^{-1}G^T(s-d)$$

where M is the updated model parameters,
M_0 is the initial model parameters,
C_N is the noise covariance matrix,
C_M is the model covariance matrix,
(s-d) is the error, observed - calculated,
G is the Jacobian matrix,
τ implies taking the transpose of the given matrix.

The perturbation term in equation (6) (the additive term to the right of M_0) is based on the error, but is constrained by the estimates of the noise and model covariances.

In order to make the inversion procedure more robust a noise model which emulates the L1 norm is employed (see Claerbout and Muir, 1973). As this model eliminates outliers from the noise model, the algorithm is generally insensitive to noise, and it favors a sparse spike, minimum entropy solution.

In addition, an event detection algorithm is used to extend the inversion beyond the maximum well depth. This extended part of the seismic logs will have reduced detail compared to the rest of the log as only the seismic data amplitudes are available to the inversion algorithm.

Final Model

As stated previously, several iterations of the linear inversion procedure may be required to obtain a final model which solves the non-linear inverse problem to within a given error tolerance. However, the final model obtained will have a greatly extended bandwidth compared to a conventionally inverted model.

A comparison between a BCI inversion and a recursive inversion is shown in Figure 3. The broadband seismic logs at a well location are shown to the right of the well log, and the recursive seismic logs are shown to the left. The recursive method used for this example is similar to the one described by Cooke and Schneider, 1983. The wider bandwidth and better character tie of the BCI logs to the well log are clearly evident.

The increased bandwidth due to the use of a priori information may be capitalized upon by reconstructing the seismic data from the inverted data using a smaller sample period and a controlled wavelet (e.g. a zero-phase 100 Hz wavelet). The choice of bandwidth used for the reconstruction being left to the discretion of the interpreter. This synthetic data set may then be displayed with the inverted seismic traces overlaid, and the data interpreted. In this way it may be possible to identify more features than could be done from the initial interpretation, and at the same time relate them to amplitude variations in the original data.

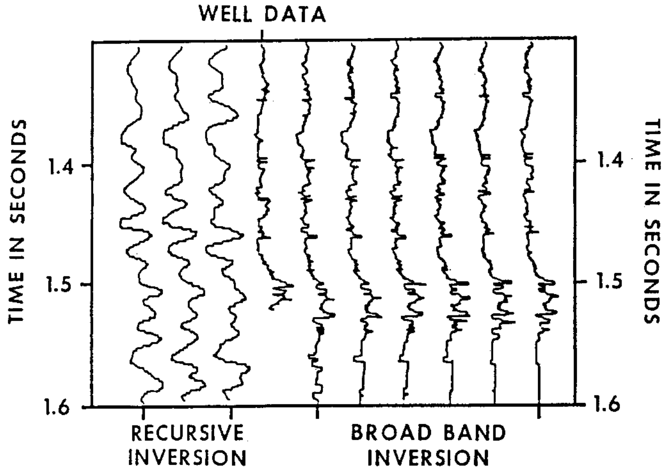

FIGURE 3
Comparison between seismic logs generated using broadband constrained inversion (BCI) and a recursive inversion technique. The BCI logs are shown to the right, and the recusive logs to the left of the well log located at the point where the comparison is made.

Conclusions

By utilizing the information from sparsely located wells it is possible to obtain broadband inverted logs from seismic data. The resultant seismic logs may then be interpreted to reveal many features not seen on the original data. The interactive workstation environment is the ideal vehicle upon which integration and manipulation of the diverse data types required may be performed.

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