Seismic Reflection Tomography

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Summary

Velocity analysis in current conventional seismic processing is beginning to appear inadequate for dealing with the increasingly complex geological structures of interest today. Tomographic imaging using individual arrival traveltimes from a given reflector may offer the prospect of reconstructing in detail an accurate velocity field above that reflector, and, by extension, the velocity field in a layered medium. Bearing in mind the likely scale of this inverse problem we formulated it as a least-squares optimisation and derived a descents-type algorithm; synthetic tests in which the reflector was assumed to be known demonstrated its potential for recovering lateral variations in particular.

However the more realistic problem in which the reflector is unknown and presumed to have structure on a scale similar to the velocity field, necessitating its inclusion in the inversion, is less tractable. As one might intuitively have expected, there are reflector depth trade-offs with near-reflector slownesses resulting in indeterminacy with severe implications for the usefulness of the reconstruction and which cannot be simply resolved. The robustness of excessively overdetermined problems suggests a multi-stage inversion in which the scale of model parameterisation is successively reduced until e.g. no further gain in resolution is achieved. Synthetic tests here suggest that while such schemes may yield significant improvements, they are dependent on the actual early-stage (large-scale) parameterisations used. Fortunately it turns out that we can use multi-stage, decreasing scale-length smoothing to achieve similar effects without such dependence.

Introduction

In current conventional seismic data processing, a weak link in achieving good images of the subsurface lies in the velocity analysis underpinning the migration processes. Despite the apparent robustness of stacking velocity calculations, failure to extract the rather different RMS velocities and the instability of the Dix formula can produce large errors in interval velocities with serious consequences for quantitative interpretation (Parkes and Hatton (1987)). Furthermore the assumptions inherent in such analysis preclude the recovery of lateral variations on a scale less than a cable-length, and so the prospects for accurate imaging, in this context, of the complex geological structures of interest today look poor.
To obtain images of smaller scale-length velocity variations, we must consider the relevant information present in individual traces. Fortunately most of this can be extracted from traveltime picks and working within the assumptions of ray theory, which considerably reduces the potential computational load. In this context traveltimes are line integrals (of reciprocal velocity, or slowness) so tomographic imaging may be feasible i.e. reconstructing velocity variations down to the scale of the receiver separation (or wavelengths) cf. Bois *et al.* (1971), Dines and Lytle (1979). Synthetic tests demonstrated the ability to recover lateral heterogeneity in the velocity field above a reflecting interface of known depth and shape. Realistically however, we must assume that the interface is at unknown depth and has structure on a comparable scale to that of the velocity, so to be consistent we must add it to the inversion. Unfortunately velocity-depth trade-offs make the resulting problem poorly-determined in a fashion crucial to the usefulness of the reconstruction; inversion schemes using regularisation in the form of gradual admission of finer-scale variations now appear to offer a sensible route to recovering an acceptable solution.

Modelling

For the purposes of the synthetic investigations undertaken so far we restricted our attention to a single layer model, divided into square cells of constant properties; at the top of each cell column is a shot/receiver site from each of which are traced rays to all of them (possibly out to some maximum offset). The spacing, which determines the cell size, is arbitrarily taken as 25 m. The reflector is specified by depth points between the cell columns, and is composed of straight sections. This parameterisation is attractive for its simplicity, particularly with regard to the inversion, but makes it impractical to trace reflected rays directly: instead, one-way traveltime functions for each shot/receiver location are created by interpolation between a set of rays traced down to the reflector using Snell's law at cell boundaries; the relevant pair of these are then added, and, by Fermat's principle, the minimum locates the reflection point and take-off angles for the (first-arrival) ray. Figure 1 demonstrates this for a model of uniform velocity field and flat, level reflector. For the $i^{th.}$ ray we may write

$$t_i = g_i(\underset{\sim}{s}, \underset{\sim}{b}) = \int_{R_i} s(\underset{\sim}{x})dl = \sum_j l_{ij}(\underset{\sim}{s}, \underset{\sim}{b}).s_j \qquad (1)$$

where l_{ij} is the length of the (straight) segment of the $i^{th.}$ ray in the $j^{th.}$ cell.

Inversion

The calculations for inversion are simplified by the fact that Fermat's principle also implies that the components of the ('Fréchet') derivative matrix of g with respect to s are simply the l_{ij} of equation (1). However the expected size of the inverse problem in real situations is likely to be such as to make direct inverse methods unattractive, even without taking

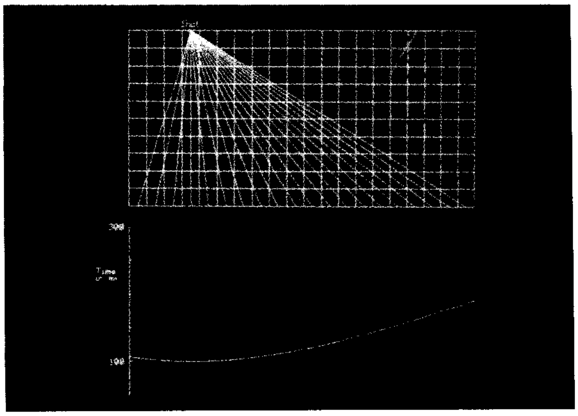

FIGURE 1
Ray-training procedure: one-way rays yield traveltime functions to be summed and minimised.

into account considerations of non-linearity (Bishop *et al.* (1985) notwithstanding). Therefore the inversion was cast in the form of a conventional least-square optimisation (e.g. Tarantola and Valette (1982)): Minimise

$$S = (\underline{t}-\underline{t}_0)^T C_t^{-1} (\underline{t} - \underline{t}_0) + (\underline{m}-\underline{m}_0)^T C_m^{-1} (\underline{m}-\underline{m}_0) \quad (2)$$

subject to $\underline{t} = \underline{g}(\underline{m})$

the model term either limiting step-lengths, with $\underline{m}_0$ the current model, or reflecting a priori information embodied in $\underline{m}_0$ with covariance C_m.

A descents-type algorithm derived using Lagrange multipliers worked satisfactorily in the initial tests where the reflector was assumed known and fixed: Fig. 2 shows that lateral variations are well-imaged, but the success at reproducing vertical variation in features depends on the offset range of the rays considered relative to the lateral scale of those features.

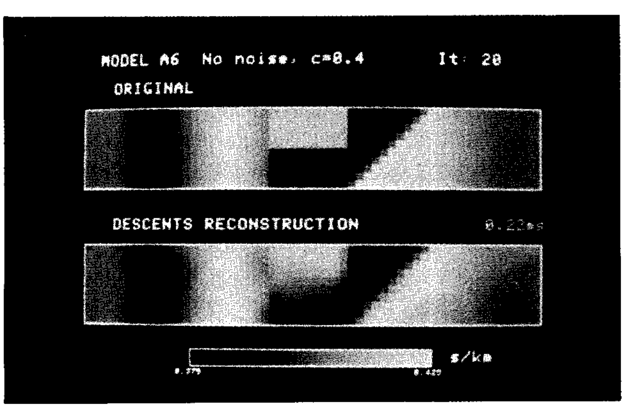

FIGURE 2
20-iteration descents reconstruction of a multi-featured slowness field above a known reflector.

However the admission into the inversion of variation in the depth parameters changes the character of the problem considerably. Figure 3 shows the attempted reconstruction of a region of lateral variation between regions of constant velocity and depth (five columns at each end, which are identical to the end columns in the picture, have been omitted here and in Fig. 4). Even with the assumption of a priori knowledge about these bounding regions and a reasonable starting estimate of the interface, and despite achieving good RMS data fit, the velocity field shows little correspondence to the original below about halfway down the model, with the interface accordingly in error.

A little consideration suggests that this is not the fault of the inversion algorithm used, but rather inherent to the problem as parameterised — there are model components with small or zero (local) singular values corresponding to trade-offs between near-reflector slownesses and depths, resulting in severe dependence on the starting model for all algorithms

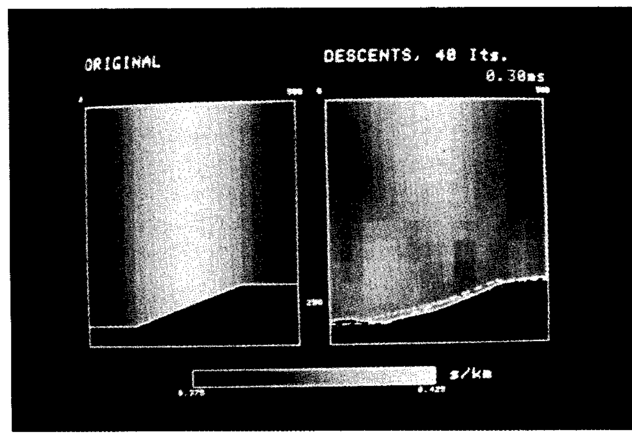

FIGURE 3
40-iteration descents reconstruction of slowness field and reflector. Starting reflector shown by short dashes.

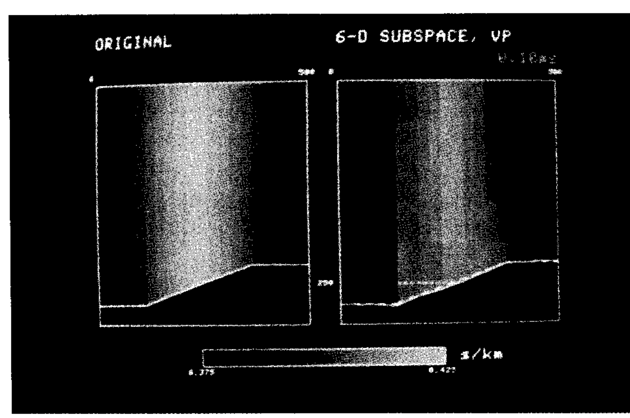

FIGURE 4
Multi-stage (variable cell-size) reconstruction of slowness field and reflector.

working with local information (gradients, curvature). Unfortunately it seems very unlikely that a starting model which yields the 'correct' answer can in general be found without almost complete knowledge of this answer.

One possible approach for dealing with this problem comes from the fact that if we look for a model to fit the same data but parameterised by only one slowness (velocity uniform across the model) and two depths (a flat, uniformly dipping reflector), corresponding to a single cell, it is overdetermined and so uniquely specified by the least-squares data fit requirement. This suggests:

(i) such a procedure may be a good way to define a simple starting model;
(ii) more generally, some kind of multi-stage inversion, starting with this and proceeding to invert for successively finer cell grids and reflector divisions, taking the result of each stage as the starting point for the next, may define a route through model space to a solution with the greatest possible resolution that can be achieved from the data. We may also suspect that this inversion plan will tend to accord with minimalist prejudices towards near-homogeneity within a layer and near-flatness of the interface.

A scheme of the kind outlined in (ii) above yielded the improved reconstruction of our test model shown in Fig. 4 — the slowness field now is a tolerable match over the whole depth range and the interface errors have been reduced considerably. We can also appreciate the near-irrelevance of the starting model due to the robustness of the first stage. Unfortunately it turns out that the results of this kind of scheme depend very strongly on the actual grid placements in the coarser stages — the success of the reconstruction shown in Fig. 4 arose in no small measure from the (inadvertent) alignment of the cell boundaries in the second stage with the reflector structure; a less favourable choice might have produced little improvement on the basic inversion.

Clearly what is required is some kind of translation-invariant regularising scheme gradually admitting shorter-scale variations (this corresponds roughly to the idea of a 'refinement' presented by Youla (1978)). One obvious possibility here would be some kind of spatial frequency filtering, but a similar effect in the model domain can be achieved by smoothing in which the scale length implicit in the model covariance matrix, C_m, is reduced in stages. This latter method has achieved similar results to Fig. 4 on that test model.

Conclusions

Although the single-layer reflection tomography problem may suffer from effective non-uniqueness of a very damaging kind, appropriate regularisation schemes may enable the achievement of satisfactory reconstructions. One undoubted factor in this is the fact that the large singular-value components correspond roughly to certain combinations of the low-frequency parts of the model spectrum, which are therefore comparatively insensitive to the non-linearity and also approximately conjugate to the other model components.

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A Review of the Development of a Digital Log Database for Petroleum Exploration

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Summary

Modern petroleum exploration, incorporating routine acquisition of geophysical logs, began in Australia in the mid 1950s.

Prior to 1976, data were recorded in analog form. By 1982, digital recording was routine for most logging units, and since 1985 few operators have accepted non-digital records.

Approximately 60% of the log data acquired to date in Australia is thus analog. Varying standards of recording practice, evolving hardware and changes in drilling practice have all affected the quality of analog records, and their ease of use. The economics of use are often highly dependent on record quality.

In 1981 Wiltshire Geological Services began systematic high fidelity conversion of analog data to digital. Our initial objectives were to develop tools for better and more efficient analysis of sedimentary sequences and basin evolution.

By 1985 the developing data set was clearly a major asset. Since then we have built the data set as rapidly as possible whilst maintaining the highest possible data conversion standards.

Resulting benefits include:
- data preservation is assured.
- data are standardized and enhanced by editing of noise.
- data reproduction is easier and cheaper.
- end-user working material is routinely tailored to specific job needs.
- worker efficiency is dramatically improved.

Intangible benefits may arise from better interpretive use of the data in exploration.