correspond to the actual intrusions. We evaluated the heat contents of the intrusive-shaped bodies using their volume and the related lavas age determined by potassium-argon method. Many hot springs occur near the intrusive-shaped bodies estimated to have high possible geothermal potential, large volume and young age. Hot springs are more sparse around those areas of low geothermal potential, small volume or old age. When we suppose that the intrusive-shaped bodies correspond to actual intrusions, the above facts can be reasonably explained. Therefore, the authors conclude there is a high possibility that the intrusive-shaped bodies extracted from the magnetic map correspond to intrusions associated with Quaternary volcanic activities in Hakkoda district.

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Bandlimited Spikiness Deconvolution (BLSD)

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Summary

The estimation and removal of the propagating wavelet in seismic data is essential to obtaining a high-resolution estimate of the earth reflectivity. A well-known approach is to assume that the reflectivity is characterized by a relatively sparse spike train and then attempt to select the deconvolution filter so as to maximize some measure of sparseness (spikiness) of the deconvolved trace. One example of this is the minimum entropy deconvolution (MED) method of Wiggins (1978). However, the spikiness deconvolution approach is not very effective in the presence of noise. The reason for this is that the best a linear deconvolution filter can do for bandlimited data in a noisy environment is to produce the reflectivity convolved with a bandlimited zero-phase waveform. Unfortunately, interference patterns generally preclude us from obtaining this output with a wide-band spikiness criterion since the desired output trace is not recognized as being spiky even though the underlying reflectivity is spiky.

In order to solve this problem, we have developed a new spikiness criterion which can be defined over a given frequency band (bandlimited spikiness). We demonstrate the usefulness of this criterion by showing how it can be used to estimate the wavelet phase for bandlimited data under the assumption that the phase is a constant, independent of frequency.

Introduction

Bandlimited spikiness

We suppose that the seismic trace satisfies the convolutional model:

$$x_t = w_t * r_t + n_t \tag{1}$$

(* denotes convolution)

where the wavelet w_t, the reflectivity r_t and the noise n_t are unknown. Let d_t be the linear deconvolution filter to be determined. The output trace is then given by:

$$y_t = d_t * x_t \ . \tag{2}$$

In spikiness deconvolution, we often try to find the deconvolution filter which maximizes the norm ratio:

$$\left(\frac{L_4}{L_2}\right)^4 = \frac{\sum_t y_t^4}{\left(\sum_t y_t^2\right)^2} \tag{3}$$

This is referred to as the varimax norm and was introduced by R. Wiggins (1978) in the method known as minimum entropy deconvolution (MED). This norm ratio is just one of the many possible measures of spikiness. Other measures include the D norm (Cabrelli, 1985),

$$\frac{L_\infty}{L_2} = \frac{\max_t |y_t|}{\left(\sum_t y_t^2\right)^{\frac{1}{2}}} \tag{4}$$

and the modified varimax norm (Ooe and Ulrych, 1979)

$$V_e = \frac{\sum_t z_t^2}{\left(\sum_t z_t\right)^2} \tag{5}$$

where $z_t = 1 - \exp(-a^2 y_t^2)$

The modified varimax norm was introduced to obviate the discrimination against small spikes observed in the varimax norm and approaches the varimax as $a \to 0$.

Unfortunately, spikiness deconvolution is not generally used by the industry since it does not appear to have a practical application. The basic problem with spikiness deconvolution has to do with the bandlimited nature of seismic data. When noise is present in the data, the best that we can do with a linear deconvolution filter is to obtain a bandlimited estimate of the reflectivity. That is, our best estimate is of the form:

$$\tilde{r}_t = y_t = p_t * r_t + n_t' \qquad (6)$$

where p_t is a bandlimited zero-phase output waveform given by:

$$p_t = d_t * w_t$$

and n_t' is the residual noise,

$$n_t' = d_t * n_t \ .$$

However, the estimated reflectivity $\hat{r}_t$ is not generally recognized as spiky via the above measures (3)–(5), even though the actual reflectivity r_t is spiky. The reason for this is the presence of interference patterns and tuning effects due to the bandlimited nature of p_t. If we attempt to reduce the tuning effects by widening the bandwidth of p_t, the residual noise increases in the low amplitude regions of the spectrum and $\hat{r}_t$ is still not recognized as spiky.

The adverse effects of tuning in spikiness deconvolution were reported by Levy and Oldenburg (1987) in their use of the varimax norm (3) to estimate and correct for the wavelet phase using a constant-phase correction. To demonstrate these effects, we have generated the synthetic reflectivity shown in Fig. 1a and a bandpass version of the reflectivity using an 8–50 Hz zero-phase Butterworth filter (Fig. 1b). Figure 2a shows the results of computing the varimax norm as a function of a constant-phase correction applied to the bandpass reflectivity. Due to tuning effects, the maximum of this curve occurs at a phase of 55° rather than zero. Although not shown here, the phase error is still significant even when the low frequencies are retained by using a 0–50 Hz filter. Thus, the tuning effects depend on both the high and low frequencies.

It follows that, in order to apply the spikiness criterion to the deconvolution of seismic data, we need to have some way of defining spikiness over a limited frequency band. To develop this approach, we first rewrite (6) in the frequency domain,

$$\tilde{R}_f = Y_f = P_f R_f + N_f' \qquad (7)$$

where

$$P_f = D_f W_f$$

Now suppose that P_f is approximately constant over a given frequency band,

$$P_f \simeq 1, \ f_\ell \leq f \leq f_u$$

If the noise is small over this band, (7) implies that

$$Y_f \simeq R_f, \ f_\ell \leq f \leq f_u \qquad (8)$$

Thus, the appropriate choice of a deconvolution filter can produce a good estimate of the reflectivity over a given band (usually, the band of highest signal-to-noise ratio). We would

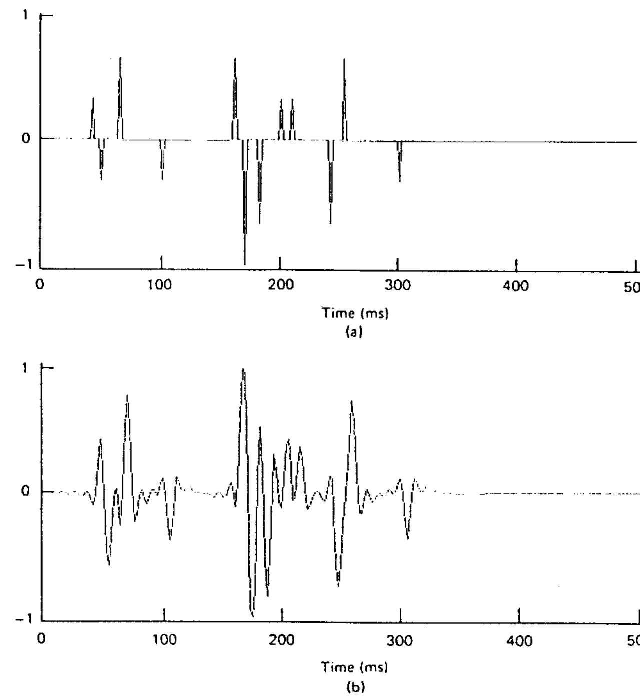

FIGURE 1
Synthetic model:
(a) Reflectivity
(b) Reflectivity convolved with 8–50 Hz Butterworth filter.

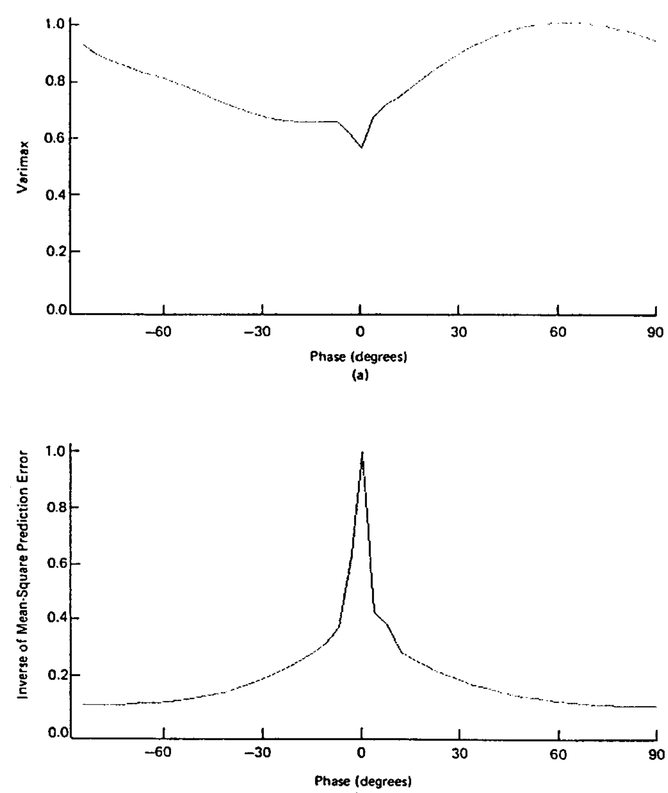

FIGURE 2
Spikiness as a function of phase rotation:
(a) Wideband using the varimax criterion
(b) Bandlimited using the inverse of the mean-square prediction error.

have a way of estimating this filter if we could measure the spikiness of y_t from the bandlimited frequency domain data Y_f, $f_l \le f \le f_u$.

In general, there are many ways of defining bandlimited spikiness. For example, any of the wideband measures (3)–(5) could be used to define a bandlimited measure via the following optimization problem:

$$\text{Find } y_t \text{ to maximize } V(y_t) \qquad (9)$$

subject to the constraints

$$Y_f = \sum_t y_t e^{-j2\pi ft}, \quad f_\ell \le f \le f_u$$

where $V(y_t)$ is any wideband spikiness measure. If ϵ is the maximum value of (10),

$$\epsilon = \text{Max. } V(y_t),$$

then ϵ constitutes a measure of bandlimited spikiness. It is clear that the solution to (9) amounts to a particular way to extrapolate the given spectrum Y_f, $f_l \le f \le f_u$. When $V(y_t)$ is the inverse of the L_1 norm, this becomes the spectral extrapolation approach of Oldenburg, Scheuer and Levy (1983).

In principle, any of the bandlimited spikiness measures based on (9) could be used to develop a deconvolution method. However, the difficult optimisation problem associated with maximising the measure ϵ with respect to the wavelet makes most of the measures impractical. There is, however, one measure that we have found to be useful for deconvolution. This measure is based on the concept of prediction filtering in the frequency domain.

Mean-square prediction error as a bandlimited spikiness measure

One approach to spectrum extrapolation involves the use of prediction filtering and was suggested by Lines and Clayton (1977) and Walker and Ulrych (1983). In this approach, a complex prediction filter is designed on the spectrum over the given frequency band $f_l \le f \le f_u$ and then used to extrapolate (predict) the spectrum outside the band. This spectrum extrapolation method suggests a bandlimited spikiness measure given by inverse of the sample mean-square prediction error; i.e.,

$$\varepsilon = 1/P_L$$

where

$$P_L = \underset{\{H\}}{\text{Min.}} \sum_f |F_f|^2 + |B_f|^2 \qquad (10)$$

and

$$F_f = \sum_{i=0}^{L} H_i Y_{f-i}$$

$$B_f = \sum_{i=0}^{L} \overline{H}_i Y_{f+i}$$

(- denotes complex conjugate)

Here F_f and B_f represent the forward and backward errors, respectively, and the indicated minimum is taken over the complex L + 1 point prediction-error filters with $H_o = 1$. Intuitively, we might expect this to be a spikiness measure since P_L is zero when Y_f consists of a sum of L (or less) complex sinusoids of the form

$$Y_f = \sum_{i=1}^{L} a_i e^{-j2\pi f\rho_i} \qquad (11)$$

where the a_i are real. However, for ϵ to be a useful spikiness measure, we must also require that the converse be true. That is, when $P_L = 0$, we want Y_f to be of the form (11). We can show that the converse holds if P_L is computed through the origin over the two-sided domain $-f_u \le f \le f_u$. Since we only have Y_f over the domain $f_l \le f \le f_u$, we need to estimate the missing data, Y_f, $0 \le f \le f_l$ as part of the spikiness measure. That is, we take

$$\varepsilon = 1/P_L$$

where P_L is computed over the domain $-f_u \le f \le f_u$ and minimized with respect to both the filter H_f and the missing data Y_f, $0 \le f \le f_l$. In practice this is done by using the 'gapfiller' method of Walker and Ulrych (1983). Figure 2b shows the results of computing this bandlimited spikiness measure as a function of a constant-phase correction applied to the bandpass reflectivity of Fig. 1b. Note that the maximum is now attained at the correct phase of zero.

Conclusions

Wideband spikiness deconvolution methods such as MED generally do not perform well in practice since they do not properly account for the bandlimited nature of seismic data. A better way to treat the deconvolution of seismic data is to design the operator based on the concept of bandlimited spikiness where spikiness is measured from the data over a limited frequency band (the band of highest signal-to-noise ratio). BLSD provides a more robust way to remove the interference effects of the wavelet and to generate an accurate estimate of the reflectivity.

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