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# Computation of Response Spectra from Adjusted Strong Motion Accelerograms

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## Summary

Because of the complicated nature of earthquake induced ground motions and the corresponding transient response of structures to such motions, the use of response spectrum has achieved wide acceptance, in the field of earthquake engineering, as a meaningful measure of the intensity of an earthquake. Thus, it is useful to investigate the application of the digital computer to characterise real earthquake motion (in the form of digitised acceleration time histories) by means of response spectra. The conceptual development of the response spectrum (which should not be confused with the ground motion spectrum) and its application to the analysis of transient oscillations in elastic systems is attributed to Benioff (1934), Neuman (1936), and Biot (1943). Its engineering significance lies in the fact that once the spectrum is known for a one-degree-of-freedom system, it is possible to compute the value of the maximum shear produced by an earthquake. Further, extension of this concept of response spectra to multidegree of freedom systems can be done using the modal superposition method of dynamic analysis. The mathematical formulation for performing response analyses of a single-degree-of-freedom system is explained.

### Single-degree-of-freedom damped system subjected to an arbitrary ground motion

Consider a mass m (which is assumed to be infinitely rigid) connected to the ground by weightless springs, such that it can oscillate (Fig. 1). Further, it possesses a spring constant k, and for the sake of analytical convenience is subject to viscous damping expressed by the constant c. In Fig. 1 it will be observed that damping is represented graphically as a dashpot. In such a viscous damping model, the damping force is proportional to the mass, while the elastic resistance force is proportional to the displacement of the mass m.

The equation of motion of such a system subjected to an arbitrary ground motion $x''_g(t)$ is:

$$m\,x''_t(t) + c\,x'(t) + k\,x(t) = 0 \qquad (1)$$

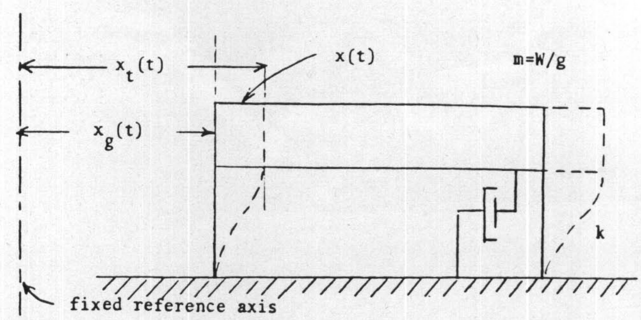


**FIGURE 1**
**Arbitrary ground motion.**

This is a second order linear differential equation with constant coefficients. Substituting $x''_t(t) = x''_g(t) + x''(t)$ in equation (1), the equation of motion becomes:

$$m\,x''_t(t) + c\,x'(t) + k\,x(t) = -m\,x''_g(t) \qquad (2)$$

where m = mass of structure
k = stiffness of structure
λ = damping ratio
ω = natural circular frequency
c = viscous damping coefficient
W = weight of structure
g = acceleration due to gravity
$x(t)$, $x'(t)$, and $x''(t)$ = Displacement, velocity and acceleration respectively of the structure relative to the ground.
$x_g(t)$, and $x''_g(t)$ = Displacement and acceleration of ground motion.

As the following relationships exist between the coefficients in equation (1)

$$\frac{k}{m} = \omega^2$$

$$c = 2m\lambda\omega = 2m\lambda(km)^{1/2}$$

Equation (2) can be written in the form:

$$m\,x''(t) + 2\lambda m\omega x'(t) + k\,x(t) = -m\,x''_g(t) \qquad (3)$$

The solution of equation (3) at rest conditions ($x_o = x'_o = 0$) is:

$$x(t) = \frac{-1}{\omega\sqrt{(1-\lambda^2)}} \int_o^t x''_g(\tau) e^{-\lambda\omega(t-\tau)} \sin[\omega\sqrt{(1-\lambda^2)}(t-\tau)] d\tau \qquad (4)$$

where $d\tau$ = Integration interval at time $\tau$. When the difference between the damped and undamped frequencies is neglected, as is permissible for the small damping ratios found in practical structures ($\lambda < 20$ per cent), this can be reduced to:

$$x(t) = -\frac{1}{\omega} \int_o^t x''_g(\tau) e^{-\lambda\omega(t-\tau)} \sin[\omega(t-\tau)] d\tau \qquad (5)$$

It should be noted that the negative sign in equation (5) can be ignored as it has no real significance in an earthquake excitation. The maximum absolute value of the integral term in equation (5) is called the spectral velocity, $S_v$.

$$S_v = \left| \int_o^t x''_g(\tau) e^{-\lambda\omega(t-\tau)} \sin[\omega(t-\tau)] d\tau \right| max \qquad (6)$$

The spectral velocity, $S_v$, is the maximum velocity experienced by a single-degree-of-freedom system when subjected to a given ground motion acceleration time history, $x''_g(t)$. The maximum response velocities, $S_v$, can be determined for a series of single-degree-of-freedom systems with different natural periods, T, and different damping ratios, $\lambda$. A plot of the response velocities against the periods, and damping ratios, of the single-degree-of-freedom systems is termed the response velocity spectrum, or simply the velocity spectrum.

In structural analysis, however, it is most convenient to work in terms of response acceleration rather than in response velocity. In this case, the maximum response acceleration, $S_a$, of a single-degree-of-freedom system subjected to a given ground motion acceleration time history, $x''_g(t)$, for small damping ratios, $\lambda$, can be determined from

$$S_a = \omega S_v \qquad (7)$$

The response acceleration, $S_a$, can also be determined for a series of single-degree-of-freedom systems with different natural periods, and damping ratios subjected to a given acceleration time-history, $x''_g(t)$. A plot of the response accelerations against the periods, and damping ratios of the series of systems are termed the response acceleration spectrum. If the acceleration time-history, $x''_g(t)$, is known, $x_{max}$, the maximum displacement of the structure can be determined from equation (5). If the velocity spectrum is known, $x_{max}$ can be determined from

$$x_{max} = Sv/\omega$$

If the acceleration spectrum is known, the maximum displacement can be determined from:

$$x_{max} = Sa/\omega^2$$

The maximum base shear $X_{max}$ in the structure can be calculated from

$$X_{max} = kx_{max}.$$

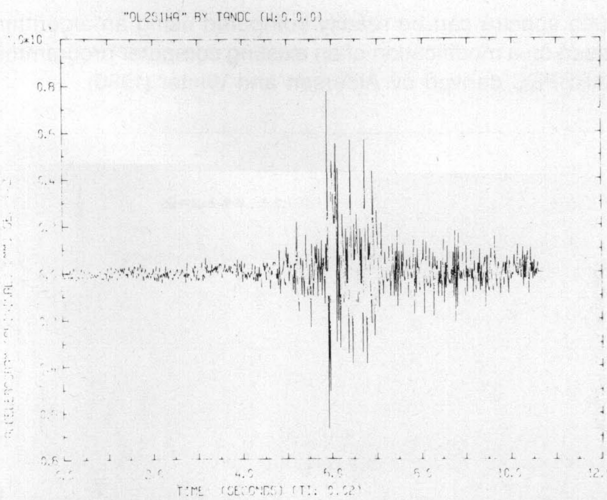

**FIGURE 2**
**N-S component of Dalton earthquake of 4th July, 1977 as recorded at Oolong (partly synthesized).**

## Computer programme for deriving response spectra from digitized strong motion accelerograms

Because typical ground motions $x_g(t)$ are often very complex, dynamic equations of motion are generally solved numerically with a computer. These equations are solved either in the form of the differential equation shown in equation (2), or the Duhamel Integral Equation given by:

$$x(t) = \frac{-1}{\omega\sqrt{1-\lambda^2}} \int_o^t x''_g(\tau) e^{-\lambda\omega(t-\tau)} \sin[\omega\sqrt{1-\lambda^2}(t-\tau)] d\tau \qquad (8)$$

where t is a point in time at which the response is being calculated, and $\tau$ is a point in time at which the acceleration impulse occurred.

Much effort has gone into the development of computer programmes for solving dynamic equations of motion in the past thirty years or so and many procedures have been developed (Milne, 1953; Berg, 1963; Cakiroglu and Ozmen, 1968; Nigham and Jennings, 1969; Wilson, Farhoomand and Bathe, 1973; Alderson and Winter, 1980).

As an algorithm for generating response spectra based on a modification of an existing computer programme SARSPEC derived by Alderson and Winter (1980), was found to be an efficient and fast running programme for deriving response spectra, it was adopted by the writer for deriving the example response spectra which appear in Figs. 3–5. These response spectra relate to the Dalton earthquake of 4th July, 1977 (Mumme and McLaughlin, 1985; Mumme, 1981). (See Fig. 2).

This programme requires input in the form of digitized accelerometer data at specified time intervals, and provides the output as acceleration, velocity and displacement response spectra for 0, 1, 2, 5 and 10% critical damping.

## Conclusions

Acceleration, velocity and displacement response spectra were derived using acceleration time-histories which have been corrected for non realistic drift with time.

Such spectra can be readily computed using an algorithm based on a modification of an existing computer programme SARSPEC derived by Alderson and Winter (1980).

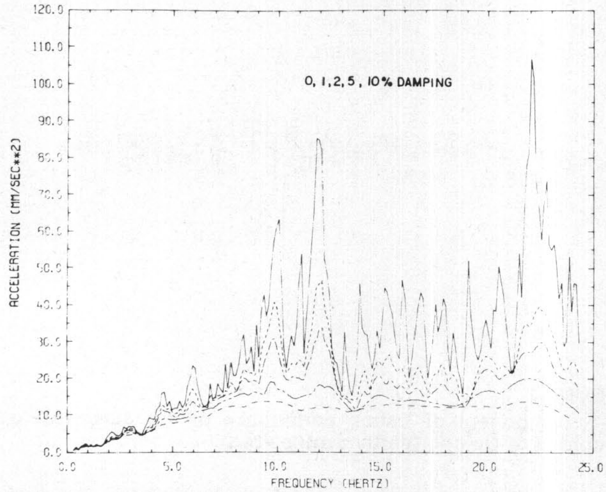


**FIGURE 3**
**Acceleration response spectra for the N-S component of the Dalton earthquake of 4th July, 1977.**

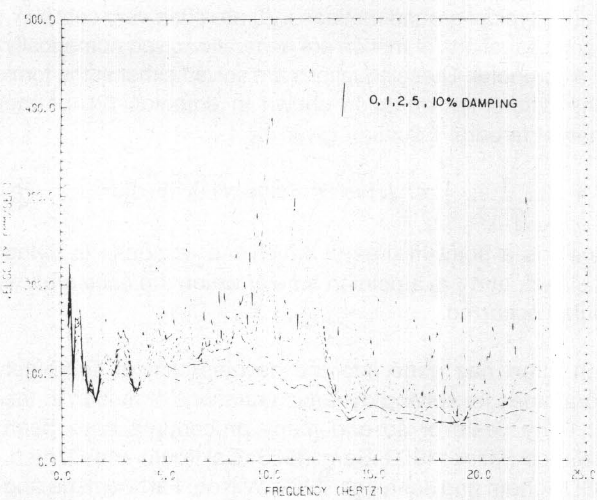


**FIGURE 4**
**Velocity response spectra for the N-S component of the Dalton earthquake of 4th July, 1977.**

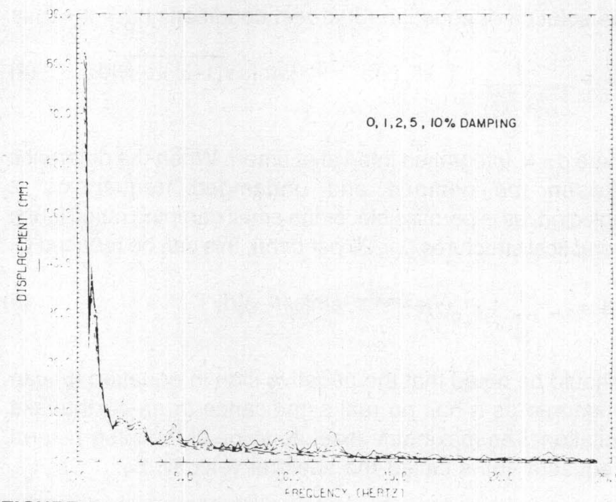


**FIGURE 5**
**Displacement response spectra for the N-S component of the Dalton earthquake of 4th July, 1977.**

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# Time Domain CSMT Method

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## Summary

CSAMT method is widely used in Japan for mining and geothermal exploration. For geothermal applications we need to study the subsurface structure as deep as 2 km or more. This shifts the CSAMT survey to a relatively lower frequency range, where the CSAMT method tends to become less reliable and less efficient. For this reason we are developing a time domain version of CSMT method at the Geological Survey of Japan. Using a controlled source excitation with a period of four seconds the waveforms of electric and magnetic fields are measured. Noisy data are rejected by visual inspection, and