hundred kilometres from the coast. The studies have closely defined these anomalous fluctuations and in particular a new study has defined the coast effect where it is a maximum over the continental slope. The results could already be used to accurately correct for temporal magnetic fluctuations occurring during regular magnetic surveys.

The anomalous magnetic fluctuations are also being used to define the electric conductivity structure of the Australian continental margin. As well as providing tectonic information, the resulting conductivity models may allow prediction of the geomagnetic coast effect at other parts of the Australian continent.

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Seismic Velocity Field Estimation — Strategies for Large-scale Nonlinear Inverse Problems

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Summary

The estimation of the seismic velocity field in two or three dimensions, by modelling the travel times of particular seismic phases or by matching observed and computed seismograms, represents a large-scale nonlinear inverse problem. The solution can be obtained by determining the minimum of a misfit function between observations and theoretical predictions, subject to some regularisation conditions on the behaviour of the model parameters. The minimisation can be achieved without the inversion of large matrices by using a search scheme based on the local properties of the misfit function. At each step in the iterative process, a subspace of a small number of directions is constructed in model space and then the minimum sought in a quadratic approximation on this set. At least two directions are required for rapid convergence. This approach is very suitable when the model parameters are of different types, since partitioning by parameter class avoids dependence on scaling.

If the model is to remain close to a reference then the regularisation term is particularly important and different types of a priori information (e.g. geological) can be introduced via the character of this term. When fit-to-data is emphasised there is the chance of finding features suppressed in a more conservative approach, but at the risk of introducing spurious detail.

Introduction

The estimation of the seismic velocity field plays a central role in many aspects of seismology, yet none of the measurements we make give the seismic velocities directly. We have to resort to indirect methods to infer the velocity parameters based on particular physical models, e.g. the recovery of interval velocities from stacking velocities depends on the assumption of some form of stratification.

When an attempt is made to reconstruct the seismic velocity field in two or three dimensions using surface or down-hole observations, we are faced with a large scale inverse problem. If we use the full seismic waveforms from many receivers, the number of data points is of the order of 10^9, and 10^5–10^6 model parameters need to be estimated to produce a full two-dimensional picture. Even if attention is concentrated on travel time picks for particular seismic phases, there will often be many thousand data values and hundreds of model parameters. In this case, the likely resolution is lower so that a coarser parametrisation of the velocity field is appropriate.

Inversion for the velocity field

The aim of an inversion procedure is to generate a set of model parameters for which the calculated values of the data points fit the observations, and also to provide information on the reliability of the model estimates. For seismic traces or travel times the functional dependence of the data on the parameters of the proposed velocity field is nonlinear (e.g. the ray paths along which the travel times are to be calculated themselves depend on the velocity distribution). Thus we seek efficient computational procedures which can cope with large problems and also deal with the nonlinearity.

We need to choose some measure of the misfit between data and theoretical predictions, and also to establish a regularisation criterion on the model parameters to prevent unreasonable behaviour. This condition will normally place limits on the deviations from a specified reference model. If the statistics of the data residuals can be estimated, then a maximum likelihood estimator of the data misfit is to be preferred to the usual sum of squares of scaled residuals. The goal of the inversion will be to reduce the data misfit to below an acceptable level by adjusting the model parameters subject to the regularisation constraints.

We now wish to minimise the data misfit. For a limited number of model parameters this can be achieved by direct search (Sambridge & Kennett, 1987) or Monte-Carlo methods (Rothman, 1986), but these techniques cannot be used effectively for many hundreds of parameters. Inversion of large matrices can be avoided by adopting a search scheme based on the local properties of the data misfit, which leads to an iterative update of the model parameters. The simplest approach is to use a single direction of search at each step (usually the gradient of the misfit function), but the rate of convergence is often slow (Tarantola, 1987). The efficiency of the procedure can be improved by the introduction of further directions so that the minimisation search is carried out in a 'subspace' of the model parameters (Kennett & Williamson, 1987). Suitable second directions are the rate of change of the gradient of the misfit function or the gradient of the regularisation term. In many cases it is undesirable to attempt to resolve fine detail at the start of the inversion, and so the procedure should be geared to the progressive refinement of scale as the fit of the proposed model to the data improves.

When the model parameters divide into classes of different types, the use of simple gradient methods mix contributions with different characters or physical dimensionality. Such problems arise, for example, in seismic reflection tomography with inversion for both the velocity field and the shape of the reflectors, and in attempts to recover both velocity and density information from reflection seismograms. The scale-dependence can be removed by once again using a local 'subspace' of the model parameters. The gradient of the misfit function has to be partitioned into parts associated with each class of parameters. The analogue of introducing the rate of change of the gradient requires the inclusion of cross-coupling between the parameters. For example, with two classes of parameters, two partial gradients and four directions from the rate of change of gradients constitute a useful six-dimensional subspace for estimating the model variation at each stage of the inversion (Kennett & Williamson, 1987).

Termination criteria

One problem with a minimisation procedure for a nonlinear functional is that the process may converge to a local minimum where the level of data misfit is not reduced below the desired threshold. In this case there may be no alternative but to start the inversion afresh from a different starting model, in the hope of coming closer to the global minimum of the misfit.

It is also possible to reach a set of model parameters whose fit to data is satisfactory but for which the regularisation conditions are not satisfied. In this case a further minimisation is needed to reduce the regularisation term to the required level. The situation is sketched in Fig. 1. The region in model space for which the fit between observations and theoretical predictions is below the prescribed threshold is indicated by *D*. The region for which the regularisation constraints are met is indicated by *R*. Those models whose parameters lie in the intersection of *D* and *R* (shaded in Fig. 1) meet both criteria and are suitable candidates for acceptance.

If the model which deviates least from the reference model is desired (in some sense the one with least new detail) then this is to be found on the edge of the region *D* within *R* and is indicated by the filled star in Fig. 1. When, however, fit-to-data is paramount, the required model will lie at the edge of region *R* within *D* and is shown by the open star in Fig. 1. Such a model will possibly include some spurious detail but might show up relevant features suppressed in a more conservative approach. A measure of the resolution of features in the constructed seismic velocity field can be obtained by looking at a sequence of models with a different balance between data fit and regularisation.

The character of the regularisation term will have a strong influence on the class of acceptable models and can be arranged to allow the inclusion of *a priori* information on the likely nature of a successful model. Such information might come, for example, from external geological input.

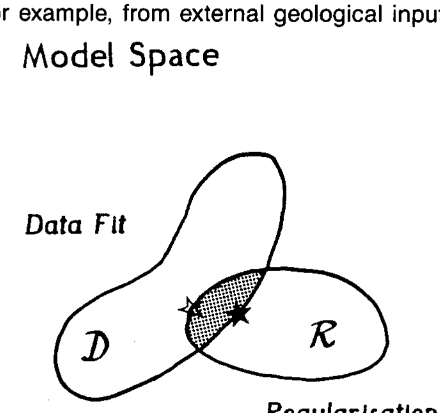

FIGURE 1
The domains *D* of acceptable models by data fit, and *R* by regularisation criterion, the shaded zone of overlap in model space meets both requirements.

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