Waves in Discontinuous Coal Seams with Absorption: Finite Difference Simulations

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Summary

The finite-difference method is a well-established tool for the forward modelling and migration of seismic wavefields in complicated geological media. One problem, however, has been that one could not include the effects of intrinsic attenuation because of numerical limitations. It has been suggested that a Padé approximation to the viscoelastic moduli turns the equations into a numerically tractable form. To achieve this approximation we suggest an approach which is based partly on physical considerations and partly on numerical curve fitting, and is superior to earlier approaches both in accuracy and computational efficiency. Our finite-difference programs for acoustic and elastic wavefields only require about twice as much computer time and storage as for a calculation without absorption. We have carried out numerical simulations of channel waves in discontinuous coal seams and have investigated the reflection and transmission responses of different types of discontinuities for models with and without intrinsic attenuation. The results show that for the most likely range of attenuation ($Q_{coal} \sim 50$–100) serious damping of the higher frequencies occurs, especially of the fundamental mode Airy phase. The differences in the reflectivities and transmissivities of different discontinuities are reduced considerably. This makes it much more difficult to detect faults and to discriminate between different fault types and throws than is expected from previous modelling of seam waves without attenuation.

Introduction

In recent years methods for the computation of complete wave fields in 2-D and 3-D inhomogeneous media have received special attention. A disadvantage of these methods like the finite-difference (FD) method has been so far that the effects of anelastic attenuation could not be included. The first attempt to incorporate realistic attenuation laws into the FD method has been made by Day & Minster (1984): an expansion of the viscoelastic modulus into a rational function of frequency turns the stress-strain relations into a differential form which is suitable for a numerical solution. Recently, Emmerich & Korn (1987) suggested a method to determine the coefficients of the rational approximation which gives fairly accurate results for *low-order* approximants. This is crucial in practice because the computational effort increases with the order of the approximation.

After a brief outline of the theory we compute Love channel waves propagating in discontinuous coal seams. This problem has been previously attacked for the elastic case (e.g. Korn & Stöckl, 1982; Aston *et al.*, 1984; Kerner & Dresen, 1985). Our computations show that the assumption of realistic attenuation in the coal has serious effects on the reflection and transmission response of coal seam discontinuities.

Theory

In the case of linear viscoelasticity the relation between stress σ and strain ϵ becomes

$$\sigma(t) = \int_{-\infty}^{t} M(t-\tau)\,\epsilon(\tau)\,d\tau \quad , \qquad (1)$$

where $M(t)$ is the inverse Fourier transform of the complex frequency-dependent modulus $M(\omega)$. This equation is useless for numerical calculations of wave propagation, because a direct numerical integration would require an immense amount of computer memory and time. If, however, the modulus is a *rational function* of ω, i.e.

$$M_n(\omega) = M_U - (M_U - M_R) \sum_{j=1}^{n} a_j\,\omega_j / (i\omega + \omega_j), \qquad (2)$$

then the stress-strain relation can be transformed into

$$\sigma(t) = M_U \left[\epsilon(t) - \sum_{j=1}^{n} \zeta_j(t)\right] \quad . \qquad (3)$$

Here, $M_U = M(\omega = \infty)$ and $M_R = M(\omega = 0)$. Each of the new variables ζ_j satisfies a first-order differential equation

$$\frac{d\zeta_j}{dt} + \omega_j\,\zeta_j = a_j\,\omega_j \left[1 - \frac{M_R}{M_U}\right] \epsilon \quad . \qquad (4)$$

Equation (3) and (4) can be readily incorporated into existing FD programs. For efficient calculations, it is crucial to keep the number n as low as possible. The problem therefore is to find accurate low order approximants of the visco-elastic modulus.

Day & Minster (1984) proposed an analytical method for determining the coefficients a_j and ω_j in equation (2). Fig. 1 (dashed line), however, shows that their method gives a poor approximation to a Q = const law.

Emmerich & Korn (1987) recognised that equation (2) represents a rheological model consisting of n parallel Maxwell bodies and that the ω_j and a_j can be interpreted as the relaxation frequencies and weight factors of these Maxwell bodies. From the frequency dependence of their moduli it becomes clear that the best choice for the ω_j is a logarithmic equidistant distribution over the frequency band of interest.

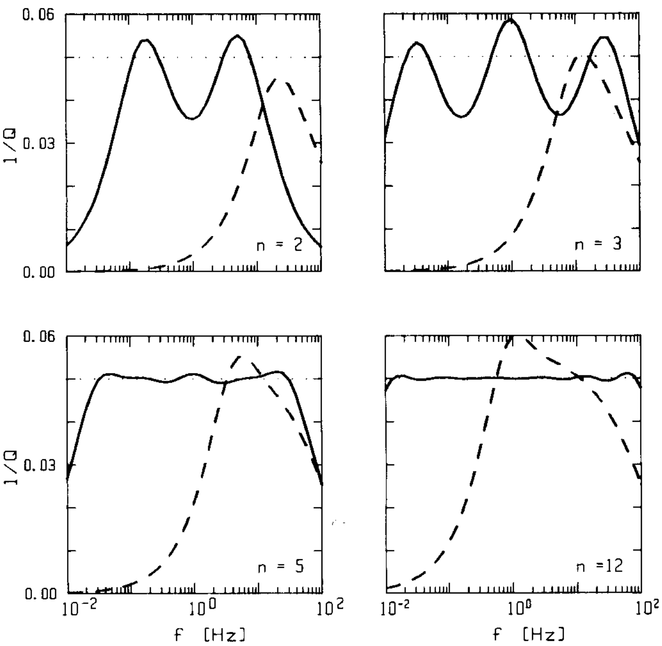

FIGURE 1
Rational approximation to the Q = Const law for Q = 20.

The weight factors can then be determined by a numerical fit to any desired Q law. The superiority of this approach is obvious from Fig. 1 (solid line). Fig. 2 compares the pulse shapes of dissipated pulses with the exact results for different dissipation times $t^* = x/cQ$ (c = velocity, x = distance). It shows that $n = 3$ should provide accurate results for all practical applications. For shorter dissipation times or low-frequency signals $n = 2$ is sufficient.

Seam wave modelling

The use of guided waves for the detection of coal seam discontinuities is a well established technique. An aim that has not been achieved so far, however, is the determination of fault structures from in-seam surveys. Numerical modelling (e.g. Korn & Stöckl, 1982; Aston *et al.*, 1984; Kerner & Dresen, 1985; Buchanan, 1986) can help in better understanding how different types of discontinuities influence the scattered wave field. These investigations have been carried out for purely elastic models. Absorption within the coal, however, may have severe implications on the detectability of faults. Most in-seam techniques rely on the Airy phase of reflected waves as indicator for the existence of a fault because this phase has the least geometrical attenuation and additionally exhibits a high reflectivity at discontinuities. On the other hand the high frequencies are more affected by intrinsic absorption than the lower frequencies of the earlier parts of the wave train. There, it is expected that the incorporation of attenuation into numerical models will change the results of the earlier studies significantly.

With our newly developed method we computed Love wave propagation in the seam models sketched in Fig. 3 Q values of 70 and 150 for coal and rock were generally assumed. The frequency dependence of the reflectivities R and transmissivities T was derived from the amplitude spectra of the reflected and transmitted waves at a distance $60d$ from the source. Source and receivers were located in the centre of the seam.

Figure 4 gives a comparison of model (1) with and without absorption. The elastic results show the well known filter effect of such a discontinuity: low frequencies are mainly transmitted and high frequencies are mainly reflected. Absorption has not much effect in the low-frequency range up to about 400 Hz m. Above this value the reflectivity and transmissivity functions are significantly affected by absorption. Their values become

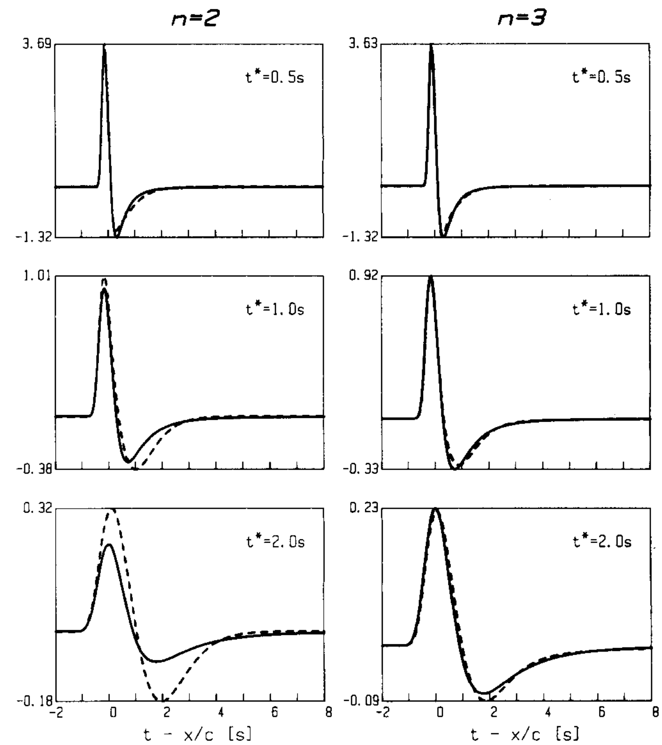

FIGURE 2
Accuracy of pulse shapes for different dissipation times, Solid: exact, dashed: approximation.

in general much smaller and the high-pass filter effect in the reflectivity has disappeared completely. The elastic model predicts pronounced differences at high frequencies if the amount of throw changes. These differences become very small in the model with absorption.

For other types of discontinuities the reflectivity and transmissivity curves look essentially similar (see Fig. 5). Model (2) with a dipping fault plane shows some dependence on the amount of throw at low frequencies but again not much difference at high frequencies. A fault zone, simulated by a low Q (model 3, solid line) takes some low-frequency energy off the reflected wave. If the fault zone additionally has a low velocity (model 3, dashed line), the reflectivity increases and transmissivity decreases considerably at low frequencies.

These results show that the presence of strong absorption makes it much more difficult to discriminate between different types of coal seam discontinuities, because the high-frequency part of the wave train which is most sensitive to small variations of the waveguide is strongly attenuated. An extension of the data analysis to frequencies well below the frequency of the Airy phase also seems not to be very promising.

Conclusions

A low-order rational approximation to the frequency dependent viscoelastic modulus is a feasible way to introduce dissipation into time-domain calculations of synthetic seismograms. Our newly developed approach is applicable to any desired Q law and yields accurate results in FD computations if computer time and memory are increased roughly by a factor of two compared to an elastic calculation.

Simulations of seam waves in discontinuous coal seams lead us to the conclusion that absorption within the coal has an important influence on the detectability of faults by means of reflection and transmission surveys. On the one hand the high-frequency part of the fundamental mode Love wave is most sensitive to small disturbances of the coal layer. On the other hand it is this high-frequency part which is most strongly affected by the anelastic properties of the coal. High attenuation within the coal therefore limits the possible range of surveys, reduces the detectability of faults and makes it almost impossible to discriminate between different fault types and throws.

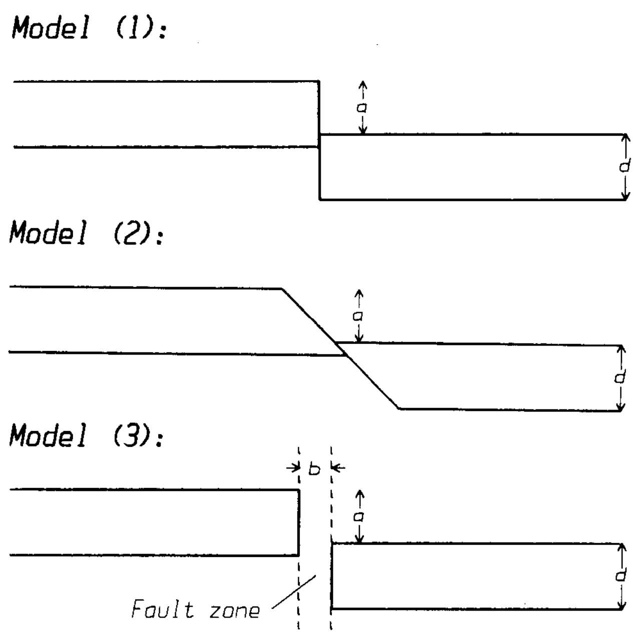

FIGURE 3
Sketch of investigated seam models.

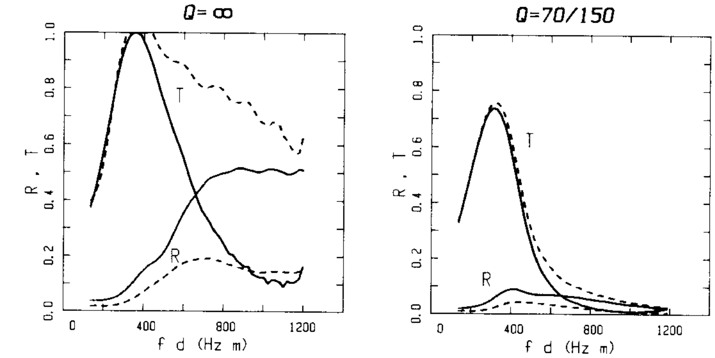

FIGURE 4
Results for model (1). Solid: a = d, dashed: a = 0.43d.

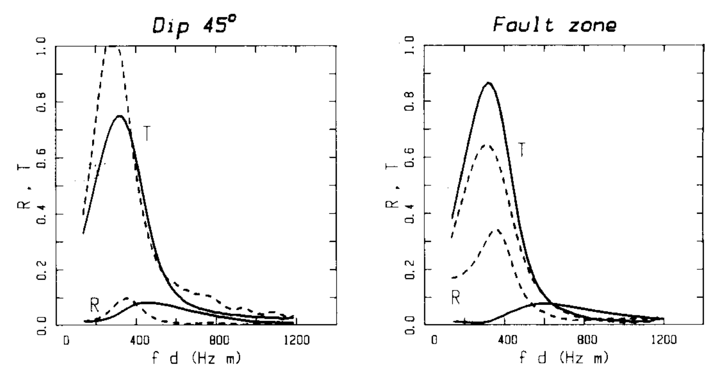

FIGURE 5
Results for models (2) and (3).

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