References

Bauer L. A. & Fleming J. A. (1915). 'Land magnetic observations 1911–1913 and reports on special researches', *in Researches of the Department of Terrestrial Magnetism, Vol. 2*, pp. 142–160. Carnegie Institution of Washington, Publication No. 175.

Dooley J. C. (1986), 'Ground control of satellite observations of the geomagnetic field', *Explor. Geophys.* 17, 4.

Milligan P. R. (1986), 'The spatial pattern of the daily magnetic variation over Australia with application to the correction of magnetic survey data', *Explor. Geophys.* 17, 24.

Petkovic J. J. (1974), 'Magnetic secular variation in the Australian region. In: Annual Summary of Activities', *BMR Rep.* 190, 51–53.

Petkovic J. J. & Whitworth R. (1975), 'Problems in secular variation in the Australian region', *EOS Trans. Am. Geophys. Union* **56**, 51–53.

von Neumayer G. B. (1869), *Results of the Magnetic Survey of the Colony of Victoria Executed During the Years 1858–1864*. Mannheim, Germany.

Winch D. E., McEwin A. J. & Barton C. E. (1986), 'Evaluation of geomagnetic reference fields', *J. Geomag. Geoelectr.* (in press).

Zmuda A. J., ed. (1971), 'World Magnetic Survey 1957–1969', *IAGA Bull.* **28**.

A model of the earth's magnetic field suitable for regional use

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A model of the earth's magnetic field is described, which is suitable for use over a limited area of the earth's surface. Coefficients for this model are derived from local observations consisting of observatory results, field measurements and aeromagnetic data. A process of deriving the coefficients has been devised which avoids problems with the stability of the solution. Finally, a system is described which allows the automated plotting of charts, based on the model. On the charts, the resolution is independent of the projection or scale being used.

Mathematical form of the model

The model used to represent the normal or main field is a set of three polynomials, first used by Reilly and Burrows (1973). The three components (X, Y and Z) are each expressed as quadratic functions of latitude, longitude and time; for example:

$$Z = a_z + b_z t + 0.5c_z t^2 + d_z xt + e_z yt + f_z x + g_z y + 0.5h_z x^2 + 0.5i_z y^2 + j_z xy$$

where x, y and t are latitude, longitude and time relative to an arbitrary position and epoch. Similar expressions are used for X and Y. This mathematical form is completely empirical. It has sufficient flexibility to model the field over a limited region. The order of the polynomial could be extended but the quadratic form has been found adequate for the New Zealand region. In principle, it might be thought better to use a spherical harmonic model, but the fitting of coefficients to such a model from observations of a limited geographical extent leads to instabilities in the coefficients if the data base is changed slightly. Although the quadratic polynomials cannot truly represent the magnetic field, it is possible to constrain them by imposing the condition that there is no vertical current flow at the earth's surface. This condition is that the vertical component of curl $\boldsymbol{B} = 0$. If the easterly component is expressed as:

$$Y = \sec\theta\,(a_y + b_y t + 0.5c_y t^2 + d_y xt + \ldots\ldots),$$

(where θ is the latitude) instead of just a polynomial, the above constraint reduces to:

$$e_x = -d_y,\ g_x = -f_y,\ i_x = -j_y,\ j_x = -h_y.$$

This set of conditions reduces the number of independent coefficients from 30 to 26.

Derivation of coefficients

There are four categories of observations available for use in the derivation of the coefficients in the model: observatory mean values; repeat stations; field observations; and aeromagnetic data from 'Project Magnet' flights. When a computerized method is being considered to derive the coefficients from the data, it is tempting to consider each observation as being equally valid, and hence assign equal weights. In practice this does not work well. The observatory results, whether monthly mean or annual mean values are used, unduly influence the spatial terms, and similarly the field observations unduly influence the time terms. As a result it has been found best from experience to divide the evaluation of the coefficients into three separate steps.

First, the coefficients of the terms dependent only on time are derived solely from the Eyrewell Observatory values. Since the major use of the model is to produce declination charts for navigation purposes, the data are weighted according to time of observation, giving reduced weight to earlier observations. The form given to the weighting function is a negative exponential, $\exp(-t/T)$. The time constant T needs to be chosen with care. Examination of the observatory mean values shows a smooth trend which can be well represented by a parabolic polynomial. Superimposed on the smooth trend are minor fluctuations, in particular some with quasi periods up to 30 or 40 years. Too short a time constant results in fitting to very recent data, resulting in a fit which is overly influenced by these fluctuations. This leads to a rapid divergence between the model and the actual field with time. If too long a time constant is chosen, the quadratic form of the time dependence is inadequate for representing the smooth trend. A time constant of at least 30 years has been found to be acceptable. The work of Reilly and Burrows used a 20 year time constant and the model was found to give a very poor extrapolation with time although the model fitted the previous decade well.

The next step is to determine the coefficients for the xt and yt terms. Observations from only the repeat stations are used in deriving these coefficients, since using such sites allows the separation of the secular variation from any local anomaly at the site. The secular variation at the Eyrewell Observatory is subtracted from the observations at the field sites. This procedure can be done either by subtracting the function $a_z + b_z t + 0.5c_z t^2$ from the field observations, or by subtracting the actual observatory values. It has been found preferable to subtract the actual observatory values, since these give a more linear residual secular variation at the field sites because the polynomial expression for the field at Eyrewell does not include the minor perturbations from a smooth trend. For each of the repeat stations a linear rate of change relative to the Eyrewell Observatory is obtained, and then these rates are used to determine the coefficients of the position-dependent secular variation terms in the polynomial expressions for the field. (i.e. d_x, e_x, d_y, e_y, d_z and e_z). Note the constraint that $e_x = -d_y$. Because of the constraint, the components cannot be dealt with separately and the function that is minimized in a least-squares fitting is:

$$(O_{xi} - C_{xi})^2 + (O_{yi} - C_{yi})^2 + (O_{zi} - C_{zi})^2$$

where O_{xi} is the ith observed value in the X component and C_{xi} is the calculated value from the polynomial expression. Several remote observatories are included with the repeat stations in this part of the analysis. The annual mean values are treated as individual observations. In the fitting procedure, the same time-weighting function is used as for the secular variation at Eyrewell. No spatial weighting function is used for these terms since the distribution of the repeat stations has been chosen to be as uniform as possible.

Finally, the data from all stations are used to derive the remaining coefficients in the polynomials. As in the determination of the position-dependent secular variation coefficients, the values at the Eyrewell Observatory are subtracted from the field station values. The residuals then consist of the sum of secular variation and position-dependent effects relative to the Observatory. The secular variation effects are removed using the coefficients just found, and the coefficients of the purely position-dependent terms (e.g. f_z, g_z, h_z, i_z, j_z) are determined from the remaining values. A least-squares fitting of the same form as that described above is used. The 'Project Magnet' aeromagnetic data are weighted by a factor of 0.5 to allow for the greater uncertainties associated with air-borne measurements. The same time-weighting function used previously is used in the determination of these coefficients. However, more care has had to be taken with the spatial weighting of observations. Stations are not distributed uniformly, and some stations provide a large number of observations while others provide only one. The object of a weighting function in this case should be to balance the contributions of regions containing a large number of observations with those from regions in which observations are sparse. The method adopted is to treat each observation at each station individually. Then to determine the weight for a particular observation, the great circle distance 'd' to every other observation is determined and the function $\exp[(d/K)^2]$ is evaluated, where K is a scale distance, typically 5°. The mean value of this function over all observations is found and the reciprocal of this mean value is the weight for the original observation.

Production of charts

The information contained in the mathematical model of the field can be conveniently displayed in the form of contour charts of the various field components and their secular variations. The mathematical form of the model lends itself to simple computer plotting of these contours. The program that has been produced to do this plotting has the feature that all calculations within it are done in terms of the Cartesian co-ordinates of the plotter itself; implying that the plotting program can handle a variety of field models, map projections and scales, but the accuracy of the finished chart is still expressed in terms of the chart itself. Contours are found where they cross the boundaries of the chart, or meridians in the case of enclosed contours. Once found, each contour is drawn as a series of straight line segments which do not depart from the true position by more than a preset limit. This limit is in terms of distance on the actual chart and not in terms of geographical distance.

In the plotting program currently in use, options are available to allow the inclusion of coastlines, a choice between Mercator and Lambert projections and either a spherical harmonic (global) or polynomial (local) field model.

Reference

Reilly W. I. & Burrows A. L. (1973). 'The Geomagnetic Field in New Zealand at Epoch 1970.5'. *NZ DSIR Bull.* **211**.