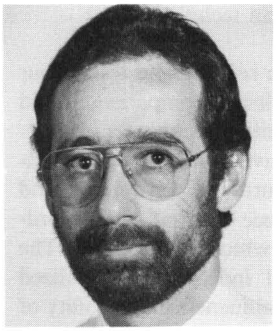

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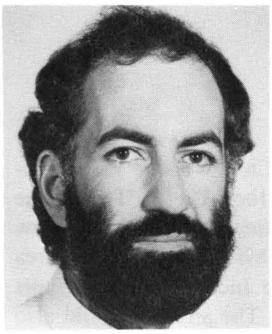

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Velocity modelling using a generalized linear inversion technique

G. R. Sutton and B. J. Moore

Introduction

Many post stack velocity modelling methods currently used in the seismic processing industry involve a top-down ray tracing procedure. In other words, rays are traced from the surface to determine the spatial locations of the next horizon of interest and the interval velocity, the procedure then being repeated as many times as there are layers of interest. A difficulty with these methods is that the error in the determination of the interval velocity is cumulative with increasing depth. In an attempt to minimize this difficulty generalized linear inversion techniques are being used (Twomey 1977; Cooke & Schneider 1983; Lines & Treitel 1984). It is this technique which is used in our approach to velocity modelling. Specifically we wish to obtain a two-dimensional interval velocity model from data available on the unmigrated stacked section.

Inversion technique

In any inversion method a knowledge of a forward model that models the generation of the data to be inverted is essential. This forward model is a function that can be either an empirical relationship or a mathematical model of the process. In our approach the forward model uses ray tracing to generate horizon times, normal moveout velocities and dips from a two-dimensional interval velocity model. Consequently, it is a nonlinear function of many variables that in general cannot be inverted analytically. The numerical technique used for the inversion is based on a Taylor series expansion of the forward model. We have

$$F(\underset{\sim}{V}_T) = F(\underset{\sim}{V}_E) + \frac{\partial F(\underset{\sim}{V}_E)}{\partial \underset{\sim}{V}_E}(\underset{\sim}{V}_T - \underset{\sim}{V}_E) + \frac{\partial^2 F(\underset{\sim}{V}_E)}{\partial \underset{\sim}{V}^2_E}\frac{(\underset{\sim}{V}_T - \underset{\sim}{V}_E)^2}{2!} + \dots$$

where $\mathbf{V}_T$ is the desired interval velocity model, $\mathbf{V}_E$ is the initial estimate of model, and F is the forward modelling function.

It is required to solve this equation for $\mathbf{V}_T - \mathbf{V}_E$ which would indicate how to correct $\mathbf{V}_E$ to make it $\mathbf{V}_T$. However, this is not possible and an approximation must be made. The approximation is to linearize the equation. We have

$$F(\underset{\sim}{V}_T) - F(\underset{\sim}{V}_E) = \frac{\partial F(\underset{\sim}{V}_E)}{\partial \underset{\sim}{V}_E}(\underset{\sim}{V}_T - \underset{\sim}{V}_E) .$$

$F(\mathbf{V}_T) - F(\mathbf{V}_E)$ is a column matrix consisting of the elements

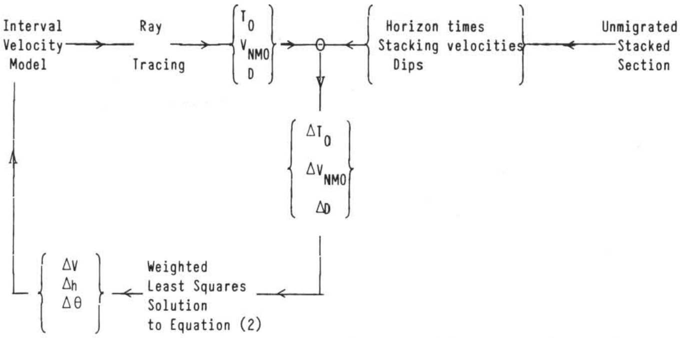
$$\Delta T_{01}, \Delta N_{NMO1}, \Delta D_1, \Delta T_{02}, \Delta V_{NMO2}, \Delta D_2 \dots$$

where T_0 is the horizon time, $\mathbf{V}_{NMO}$ is the normal moveout velocity, and D is the dip.

$\mathbf{V}_T - \mathbf{V}_E$ is a column matrix consisting of the elements

$$\Delta V_1, \Delta h_1, \Delta\theta_1, \Delta V_2, \Delta h_2, \Delta\theta_2 \dots$$

where $\mathbf{V}$ is the interval velocity, h is the depth, and θ is the slope. The iterative inversion technique is best understood by considering the following schematic:



(1) An initial estimate (guess) is made of the interval velocity model. Experience indicates this can be quite crude.

(2) Using our forward model (ray tracing techniques) T_0, $\mathbf{V}_{NMO}$ and D are determined for each horizon.

(3) These are compared with horizon times, stacking velocities and dips from the unmigrated stacked section.

(4) The differences ΔT_0, $\Delta \mathbf{V}_{NMO}$ and ΔD are used in a weighted least squares approach to solving eqn (2) for $\Delta \mathbf{V}$, Δh, $\Delta\theta$.

(5) $\Delta \mathbf{V}$, Δh, $\Delta\theta$ are used to perturb our initial estimate of the interval velocity model.

(6) Procedure is repeated until an error measure based on ΔT_0, $\Delta \mathbf{V}_{NMO}$, ΔD is within acceptable bounds.

Application of inversion technique

Esso Australia Ltd has made available the 'Sunfish' velocity model to permit testing of the inversion technique. This model was produced by Esso in 1982 to test the Geophysical Service Inc. SPACVELS software. The results of applying our inversion technique to the 'Sunfish' data will be presented at the conference.

Advantages/disadvantages of inversion technique

Much more work needs to be done in refining and testing our inversion technique before any conclusive comparison could be made with existing velocity modelling techniques. Nevertheless, our approach does seem to have some obvious advantages. Firstly the ability to use an accurate forward model and secondly the ability to weight each piece of input data according to the degree of accuracy with which it is known. The disadvantages of our approach and indeed all generalized linear inversion techniques relate to uniqueness and stability of the solution. To date these have not proven to be as difficult to deal with as we expected.

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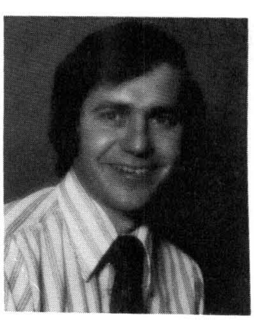

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