Bull. Aust. Soc. Explor. Geophys. (1983) **14**, 22-23

Note

Computation of the Electromagnetic Response of a Layered Earth to a Vertical Coaxial Loop System

S. S. Rai

National Geophysical Research Institute
Uppal Road, Hyderabad 500 007
India

Key words: electromagnetic, prospecting, loop system, vertical, coaxial, layered earth, response

Abstract

The electromagnetic (EM) mutual coupling ratio for a vertical coaxial loop system over a layered earth is expressed as a Hankel transformable integral. This expression facilitates the computation, using an available digital linear filter, for the response of either a perpendicular or a vertical coplanar loop configuration, or a Schlumberger electrode configuration. Such computations may be carried out on a programmable pocket calculator.

Introduction

The mutual coupling ratio (Z/Z_0) for a vertical coaxial loop system (such as designated T100, L100, R100 by Parasnis (1970)) over a layered earth is given by (Koefoed *et al.* 1972; Verma 1973)

$$Z/Z_0 = 1 - \frac{r^2}{2} \left[\int_0^\infty \lambda R(\lambda) J_1(\lambda r) d\lambda - r \int_0^\infty \lambda^2 R(\lambda) J_0(\lambda r)\, d\lambda \right] \quad (1)$$

where J_0, J_1 are Bessel functions of order 0 and 1, $R(\lambda)$ is the electromagnetic kernel over a layered earth (Koefoed 1972), and r is the coil separation.

Computation of Z/Z_0 requires the evaluation of two Hankel transforms separately. Verma (1973) used the filters for perpendicular and horizontal loop systems to evaluate the two Hankel integrals in equation (1). This procedure had to be followed as Z/Z_0 could not be expressed as a single Hankel integral. However, after showing its equivalence with the radial dipole-dipole electrode system, Das & Verma (1981) used the radial dipole-dipole filter to compute equation (1). The application of resistivity filters to compute equation (1) is found to produce an error of about 10 ppm, which is much above the common airborne EM system field recording level of 1 ppm. This level necessitates the computation of EM response using EM filters only, because of their greater accuracy. In this note, we express equation (1) as a single Hankel integral which allows us to use a single digital linear filter instead of a combination of two.

Modified expression for Z/Z_0

Following Patella (1980) equation (1) can be written as

$$Z/Z_0 = 1 - r^2 \int_0^\infty \lambda \bar{R}(\lambda)\, J_1(\lambda r)\, d\lambda \quad (2)$$

$$\text{where } \bar{R}(\lambda) = R(\lambda) + \frac{\lambda}{2} \frac{\partial R(\lambda)}{\partial \lambda} \quad (3)$$

The integral in (2) is a Hankel integral. Introducing the changes of variables $\ln r = x$ and $\ln (1/\lambda) = y$, equation (2) assumes the form

$$Z/Z_0 = 1 - \int_0^\infty e^{2x-2y} \left[-\bar{R}(y)\, J_1(e^{x-y})\right] dy \quad (4)$$

The integral in equation (4) (to now be denoted INT) may alternatively be written as

$$INT = e^{-x} \int_0^\infty e^{-y}\, \bar{R}(y) \left[-e^{x-y} J_1(e^{x-y})\right] dy \quad (5)$$

$$= \int_0^\infty R(y) \left[-e^{2x-2y} J_1(e^{x-y})\right] dy \quad (6)$$

Thus the integral in eqn (4) may be evaluated either expressing it as eqn (5) or eqn (6), which enables us to use directly the digital linear filters for either perpendicular or vertical coplanar coil systems. The filter function in eqn (5) being the same as the one for Schlumberger electrode configuration makes it possible to compute the ratio Z/Z_0 on a small pocket calculator using Ghosh's filter. However, the possibility of designing a new set of filter coefficients for vertical coaxial loop systems also exists.

Remark

In arriving at equations (2) and (3) from (1) we transform a J_0 function to a J_1 function using the relation (Gray & Mathews 1966)

$$\frac{d}{dx} \left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}(x)$$

and in the process the kernel function R (λ) is also differentiated. The above operation is exercised in the λ domain.

Electrical methods of prospecting involve either a J_0 function or a J_1 function in the response computation.

Using the derivative of a kernel function it is therefore possible to establish the similarity in the above two classes of electrical prospecting techniques.

Conclusion

The mutual coupling ratio for a vertical coaxial loop system over a layered earth is shown to reduce to a single Hankel transformable integral. This reduction facilitates the use of available filter coefficients for either perpendicular or vertical coplanar loop configurations, in place of the use of two different filters (Verma 1973).

Acknowledgments

Acknowledgment is due to Drs S. K. Verma and U. C. Das for discussion and review of the original manuscript.

References

Das, U. C. & Verma, S. K. (1981), 'Versatility of digital linear filters used in computing resistivity and EM sounding curves', *Geoexploration* **18**, 297-310.

Gray, A. & Mathews, G. B. (1966), *A Treatise on Bessel Functions and Their Applications to Physics,* Dover Publ. Inc., New York.

Ghosh, D. P. (1970), 'The application of digital linear filter theory to the direct interpretation of geoelectric resistivity measurements', Doctoral thesis, Technological Univ., Delft.

Koefoed, O., Ghosh, D. P. & Polman, G. J. (1972), 'Computation of type curves for electromagnetic depth sounding with a horizontal transmitting coil by means of a digital linear filter', *Geophys. Prosp.* **20**, 406-420.

Parasnis, D. S. (1970), 'An elegant, universal nomenclature for electromagnetic moving source-receiver dipole configuration', *Geophys. Prosp.* **20**, 406-20.

Patella, D. (1980), 'The quantitative interpretation of dipole soundings by means of the resistivity transformation function', *Geophys. Prosp.* **28**, 956-60.

Verma, R. K. (1973), 'A feasibility study of electromagnetic depth sounding methods', Doctoral thesis, Technological Univ., Delft.

(Received 24 January 1982; revised 12 July 1982)