Short Note

Conductivity and Frequency Effects in Measurements of Susceptibility

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The susceptibility of a rock specimen in relatively low fields may be measured by comparing the changes in an inductively coupled circuit when the specimen and a similarly sized standard of known susceptibility are introduced into the coupling space. It is important to know how truly such measurements represent susceptibility and to what extent the conductivity of the specimen and the excitation frequency of the instrument affect the accuracy of measurement.

Transformer Bridge of Ridley and Stone

Consider as a specific example the transformer bridge developed at the CSIRO Division of Mineral Physics by B. Ridley and D. Stone, following the principle of Collinson and Molyneux (1967). Susceptibility may be measured over the range 3×10^{-7} to 20 cgs ($12\pi \times 10^{-7}$ to 80π S.I.) at an operating frequency of 217 Hz, thus permitting the measurement of specimens ranging from quartz (-0.95×10^{-6}) to pure magnetite ($\sim$0.4 cgs).

The bridge consists of pairs of primary and secondary coils wound toroidally on circular magnetic cores, with identical gaps cut to receive the specimen, together with the exciting and measuring electronic circuitry. There are facilities to measure both the real and imaginary components of the secondary current but the interpretation of these in terms of conductivity of the specimen is not simple.

When a homogeneous cylindrical specimen is placed with its axis parallel to the gap, the susceptibility is measured parallel to this axis.

Skin Effect and Depth of Penetration of the Specimen

In default of a solution to the Helmholtz equation to fit the proposed boundaries, simpler models have been studied with less rigour and an empirical investigation made of part of the range of operation of the bridge.

In materials of moderate to high conductivity the conduction current is dominant at audio frequencies and the displacement current may be neglected. It is then possible to use the skin depth, the depth at which a plane wave is attenuated to $\frac{1}{e}$ of the incident amplitude upon an infinite plane sheet, as an indicator of the probable shielding of the interior of a specimen by eddy currents in the outer shell. If the skin depth is greater than the radius of the core (usually about 1 cm), we conclude that eddy current shielding is not significant. This argument provides an upper bound to the conductivity-permeability product for which this method can be used.

In Table (1) skin depth δ is presented ($\delta = [2/\mu\,\sigma\,\omega]^{1/2}$) for some rock types of interest. Some values for resistivity and hence conductivity σ have been taken from Parkhomenko (1967) and Parasnis (1956). Estimates for permeability μ are based on the magnetite content, or its equivalent, and the upper limit for mass susceptibility of $80{,}000 \times 10^{-6}$ emu/g (414,400 cgs volume susc.) for magnetite given by Rösler and Lange (1972), while those for hematite are from Berkman and Ryall (1976). The angular frequency ω has been calculated for the operating frequencies of 217 Hz and 10 kHz.

It is clear that the resistivity of pyrrhotite type rock could be decreased by as much as 25 times to 4×10^{-7} Ωm reducing the skin depth at 217 Hz to 18 mm, before the shielding effect becomes serious.

Empirical Results

In Figure 1 the measured values of susceptibility are plotted against magnetite content for two suites of fabricated specimens; the first were non conducting, they were made from plaster of paris and crushed magnetite from Biggendon, the second lot were made conductive by using magnetite with graphite and copper sulphate. The ranges of variation of mass in both series of specimens are similar (about 16-22 g), corresponding to ranges of magnetite content of 0.1-33% mass, and 0.02-12% volume. The resistance between discs of copper held against opposite ends of the specimens is very dependent upon the pressure on the specimen. The results of the initial measurements of this resistance and those of susceptibility are presented in Table 2.

The bulk susceptibility facility of the Digico installation (which operates at 10 kHz) at the Division of Mineral Physics, was also used to measure the suscepibility of those specimens in the two suites within its range, and these data are also presented in Figure 1 and Table 2.

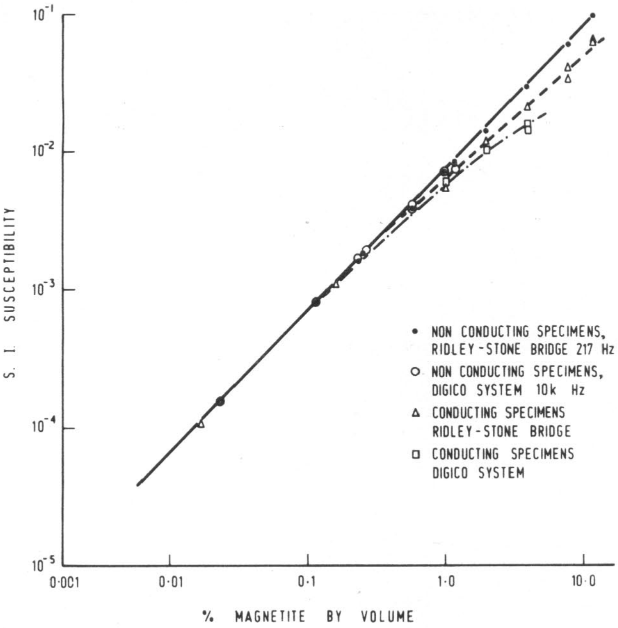

FIGURE 1

Apparent susceptibility as a function of the magnetite content for fabricated specimens with equal volumes

TABLE 1

Skin depth for plane waves in various rocks

	Resistivity ohm metres		Susceptibility χ SI	Relative Permeability $\mu_r = \mu/\mu_o$	Skin Depth δ in metres for 217 Hz	10 kHz
Copper Pyrite Ores 40% to 60% pyrite						
Low Limit Resistivity	0.3	*(1)	0	1.0	18.7	2.75
High Limit Resistivity	3.0	*(1)	0	1.0	59	8.7
Pyritic Copper Sulphur Ore						
90%	10^{-4}	*(1)	0	1.0	0.34	0.05
60%	10^{-3}	*(1)	0	1.0	1.1	0.16
Pyrrhotite						
Low Resistivity	10^{-5}	*(2)	0.35	1.35	0.09	0.01
High Resistivity	10^{-3}	*(2)	0.35	1.35	0.93	0.14
Iron Ores						
60% Fe_3O_4, and Secondary Iron Minerals	4.5	*(3)	3	4	36	5.3
Hematite Ores						
from Tennant Creek	590	*(4)	0.006	1.006	827	122
range from to	250		0.14	1.14	506	75

*(1) Parasnis, quoted in Parkhomenko (1967) p 101 and Parasnis (1956) Table 1
*(2) Parkhomenko (1967) p 102;
*(3) Parkhomenko (1967) p 106;
*(4) Berkman & Ryall, (1976) p 244.

TABLE 2

Apparent susceptibility for fabricated, non-conducting and conducting, specimens of varying magnetite content

Non-Conducting Artificial Specimens				
Specimen Label	Magnetite Content kg × 10^{-3}	% Magnetite (Volume)	S.I. 217 Hz (Ridley-Stone) × 10^{-6}	Susceptibility at 10 kHz (Digico) × 10^{-6}
BS6	0.0144	0.024	155	150
BS7	.0144	0.024	159	155
BS10	.0726	0.117	800	815
BS1	.1450	0.235	1590	1640
BS3	.1456	0.235	1560	1590
BN3	.1597	0.259	1800	1850
BN4	.1594	0.258	1780	1850
BS4	.3625	0.588	3850	3850
BS5	.3627	0.588	3430	3960
BN11	.6136	0.995	6870	6900
BS2	.7250	1.175	7820	7420
BS8	.7253	1.176	8240	8050
BS9	.7245	1.175	7960	8050
BN12	1.2261	1.988	13700	
BN13	2.4519	3.976	28800	
BN14	4.9023	9.950	57500	
BN15	7.4012	12.003	93300	

Conducting Specimens						
Specimen Label	Magnetite Content kg × 10^{-3}	% Magnetite (Volume)	Conductivity Siemens/Metre	Resistivity ohm m	S.I. 217 Hz (Ridley-Stone) × 10^{-6}	Susceptibility at 10 kHz (Digico) × 10^{-6}
BNC10	0.010	0.016	0.002	500	104	
BNC9	0.100	0.162	.003	330	1170	(A)
BNC6A	0.6126	0.993	0.08	12	5350	5600
BNC6B	0.6126	0.993	.28	3.6	5350	5620
BNC5A	1.225	1.987	.52	1.9	11900	10100
BNC5B	1.225	1.987	.4	2.5	10800	9900
BNC5C	1.225	1.987	.4	2.5	11500	10300
BNC4A	2.450	3.972	.35	2.9	19700	15500
BNC8	2.450	3.972	.68	1.5	20200	14500
BNC3A	4.900	7.946	.6	1.7	34600	(B)
BNC3B	4.900	7.946	.7	1.4	38800	
BNC2	7.400	12.000	.5	2.0	61700	
BNC7	7.400	12.000	5.3	0.19	60000	

(A) Specimen not measured because instrument noise was too great.
(B) Not measured. Above range of instrument

Discussion

It is quite difficult to separate the effects of conductivity and susceptibility in any single measurement of susceptibility, although it is clear that reducing the frequency at which measurement is made increases the range over which measurements can be interpreted with confidence. (In this respect the balanced bridge is at a distinct advantage to the Digico instrument which operates at 10 kHz.) Increasing the quantity of magnetite in a graphitic and copper sulphate specimen causes increases in both susceptibility and conductivity, but in these tests there were difficulties in achieving identical grain size distributions. Thus the 10% decrease in susceptibility between specimens BNC2 and BNC7 with equal quantities of magnetite, apparently caused by an increase in conductivity of an order of magnitude, is hardly conclusive since a 10% increase in the conductivity in specimen BNC3A,-B has an opposite effect that may well be due to a difference in magnetic grain size and distribution. The magnetite used was pure and came from Biggendon.

However, besides defining the conductivity-susceptibility domain over which the measurement of susceptibility is substantially independent of conductivity. Figure 1 suggests the use of a non-linear calibration-correction to be applied in the high conductivity region.

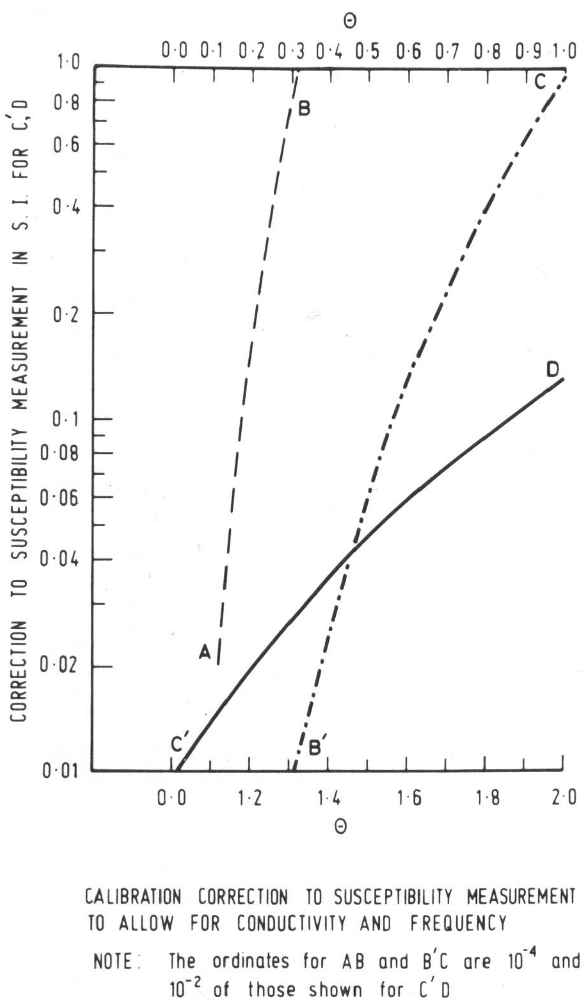

FIGURE 2

Calibration correction to susceptibility measurement to allow for conductivity and frequency. Note: The ordinates for AB and B'C are 10^{-4} and 10^{-2}, respectively, of those shown for C'D.

Based on the theory of a conducting sphere as treated by Wait (1951) and Ward (1953), a theoretical curve for the correction has been produced for the limiting case of conducting but 'non-magnetic' rocks, in terms of a dimensionless parameter $\Theta = a[\mu\ \sigma\ \omega]^{1/2}$.

Thus at an operating frequency of 10 kHz, for a radius a = 0.02 m, permeability $\mu = 4\pi \times 10^{-7}$ henrys/m, conductivity $\sigma = 10^5$ siemens/m ($\rho = 10^{-5}$ Ωm) angular frequency $\omega = 2\pi \times 10^4$ rads/s, Θ is 1.8 and a non magnetic specimen would have an apparent negative susceptibility due to conductivity of 0.09 SI (7200×10^{-6} cgs). The advantage of low frequency measurement is again exemplified by considering the same specimen at 217 Hz operating frequency. Then Θ = 0.26 and the conductivity effect is reduced to an off-set of 44×10^{-6} SI (3.5×10^{-6} cgs) which is negligible.

For many of the rock types of interest to magnetic interpretation the assumption $\mu=\mu_o$ will be invalid e.g. for pyrrhotite ore $\mu_r = \mu/\mu_0$ typically is 1.02 and may range to 1.35. However, the effect of conductivity is most serious for non-magnetic specimens and the calibration curve given therefore provides an upper limit to the necessary correction. From the form of the curves given by Ward for high frequency measurements of conductivity it is clear that Figure 2 provides a reasonable approximation for correction of susceptibility measurements for rocks of susceptibilities 0 – 1.3 SI (0 – 0.1 cgs).

References

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Acknowledgements

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