Paper

The Response of a Coincident Loop Transient Electromagnetic System above a Uniform Earth

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The response of a coincident horizontal loop transient electromagnetic (TEM) system is expected to decrease as the loop is lifted off the earth. This effect is calculated for a uniformly conducting earth for a range of loop heights, ground conductivities, loop sizes, and delay times. Within the assumptions of the model, the calculations are exact and valid for any height.

It is shown that signal attenuation with respect to height does not preclude the use of a helicopter TEM system. It is also shown that for ground TEM surveys where the loop is held off the ground because of brush, the maximum attenuation that could be expected from a uniform half space is less than 20%.

Introduction

As has been pointed out recently by Lee (1978), most of the type curves for the interpretation of transient electromagnetic (TEM) data for a coincident loop system assume that the loop is lying directly on flat ground. There is a question as to what attenuation of response is produced as the loop is lifted off the ground. The answer to this question is not only useful in evaluating the effect of the loop being strung over bushes but also in evaluating the potential of helicopter TEM systems.

Lee (1978) attacked the problem for small heights (where small was not defined) by calculating a first order correction to the expression for a loop on the ground. This approach, however, gives no indication of how correct the approximation is or for which situations it can be used. (The figure in the paper referred to evaluates the accuracy of an approximation to an approximation. It bears no relation to signal attenuation with respect to loop height or even how good the 'exact' approximation is.) Basically, the information content of a strictly first order approach is to determine the validity of the original expression in the absence of corrections.

The formulation below is valid for any height good to any specified accuracy. Three assumptions are made:

(1) the excitation voltage is a step function; i.e. it can be turned off in zero time;

(2) the response of square and 'near square' loops will be the same as that of a circular loop of the same area;

(3) the loop wire is filamentary (of zero cross-sectional area).

Derivation

From Morrison *et al.* (1969) the 'frequency domain' electric field, E(p,h), induced in a coincident transmitter-receiver loop configuration (of radius a) h metres above a uniform half space of conductivity σ, can be expressed as:

$$E(p,h) = \frac{p\mu I a}{2} \int_0^\infty \left[\frac{\lambda-S}{\lambda+S} e^{-2\lambda h} + 1\right] J_1^2(\lambda a)\, d\lambda \qquad (1)$$

where p = Laplace transform variable corresponding to $i\omega$ for $\exp(i\omega t)$ time variation

I = transmitter current

$\mu = 4\pi * 10^{-7}$

$J_1(\lambda a)$ = Bessel function of first kind

$S = \sqrt{\lambda^2 + \mu\sigma p}$

To obtain the voltage response per ampere, $V_E(p,h)$, divide by the current, I, and integrate the field around the loop which gives a factor $2\pi a$. The transform of the voltage response of the earth, V(p,h), is the product of $V_E(p,h)$ with the Laplace transform of the source function (p^{-1} in the case of step function excitation).

$$V(p,h) = \frac{2\pi\mu a^2}{2} \int_0^\infty \left[\frac{\lambda-S}{\lambda+S} e^{-2\lambda h} + 1\right] J_1^2(\lambda a)\, d\lambda \qquad (2)$$

As in Lee and Lewis (1974), the terms in brackets can be rewritten as

$$\frac{\lambda-S}{\lambda+S} e^{-2\lambda h} + 1 = \frac{2\lambda}{\lambda+S} e^{-2\lambda h} - (e^{-2\lambda h} - 1)$$

The term at the right in parenthesis is constant in p. Thus its inverse Laplace transform will be a delta function at time = 0. Hence this term has no effect on the decay curve and can be dropped. The time domain response of the remainder is

$$V(t,h) = 2\mu a^2 \pi \int_0^\infty \lambda \mathcal{L}^{-1} \left\{\frac{1}{\lambda+S}\right\} e^{-2\lambda h} J_1^2(\lambda a)\, d\lambda \qquad (3)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{\lambda+S}\right\} = \frac{1}{\sqrt{\sigma\mu}}\ \mathcal{L}^{-1}\left\{\frac{1}{\left(\frac{\lambda^2}{\sigma\mu}+p\right)^{1/2}+\frac{\lambda}{\sqrt{\sigma\mu}}}\right\}$$

$$= \frac{1}{\sqrt{\sigma\mu}} e^{-\lambda^2 t/\sigma\mu}\ \mathcal{L}^{-1}\left\{\frac{1}{\sqrt{p}+\sqrt{1/\sigma\mu}}\right\}$$

$$= \frac{1}{\sqrt{\sigma\mu}} e^{-\beta^2 t}\left[\frac{1}{\sqrt{\pi t}} - \beta e^{+\beta^2 t}\,\mathrm{erfc}\,(\beta\sqrt{t})\right]$$

(Churchill, 1958)

where $\beta = \frac{\lambda}{\sqrt{\sigma\mu}}$

erfc $(\beta\sqrt{t})$ = the error function $1 - \mathrm{erf}\,(\beta\sqrt{t})$

Next change the integration variable to ζ.

Let $\zeta = \lambda a$

H = normalised height = h/a

T = normalised time = $t/\sigma\mu a^2$

$$V(T,H) = \frac{2\sqrt{\pi}}{\sigma a\sqrt{T}}\int_0^\infty e^{-2\zeta H}\left[e^{-T\zeta^2} - \sqrt{\pi T}\,\zeta\,\mathrm{erfc}\,(\sqrt{T}\zeta)\right] * J_1{}^2(\zeta)\,\zeta\,d\zeta \qquad (4)$$

This integral can be evaluated using the following approximation

$$\mathrm{erfc} = 1 - \mathrm{erf} = (a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4 + a_5\eta^5)e^{-y^2}$$

$y = \sqrt{T}\,\zeta$

$\eta = (1 + py)^{-1}$

P = 0.3275911

a_1 = 0.254829592

a_2 = −0.284496736

a_3 = 1.421413741

a_4 = −1.453152027

a_5 = 1.061405429

This expression (Abramowitz and Stegun, 1964, p.299, 7.1.26) has a relative error $< 1.5 * 10^{-7}$.

An expression of similar accuracy for the Bessel functions can be found in Abramowitz and Stegun (p.370, 9.4.4 and 9.4.6). The numerical evaluation of the integral is facilitated by changing the variable of integration once more.

$$g = 1 - e^{-\zeta}$$

$$d\zeta = \frac{dg}{1-g}$$

This reduces the interval of integration to 0 – 1. Finally,

$$V(T,H) = \frac{2\sqrt{\pi}}{\sigma a T}\int_0^1 yJ_1{}^2(\zeta)\,e^{-(2\zeta H + y^2)}$$

$$* \left[1 - \sqrt{\pi}\,y\,(a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4 + a_5\eta^5)\right]\frac{dg}{1-g} \qquad (5)$$

This integral can be efficiently evaluated using the method of Patterson (1968) applied to Gaussian quadrature.

Results

Logarithmic decay curves of normalised voltage as a function of normalised time are shown in Fig. 1†. Normalised voltage (dimensionless), VN, is defined as $(\sigma a) *$ (volt/amp). Normalised time, T, is as previously defined ($T = t/\sigma\mu a^2$). The plot extends over ten decades of T which may seem extreme. However if t = 0.1 ms; σ = 6 s/m; and L (length of side of square loop) = 200 m, then T = 0.001. At the other end, t = 160 ms, $\sigma = 10^{-4}$ s/m, and L = 20 m gives $T = 10^7$. Curves A, B, C, D and E are the VN for H (loop height/loop radius) = 0, 0.1, 1, 10 and 100 respectively.

Fig. 2 shows a specific example for those interested in helicopter applications for single loop TEM. Others can be constructed from the curves in Fig. 1. The response factor is defined as the ratio of the voltage received at altitude to that which would be received if the loop were on flat ground. The example shown assumes a helicopter flying at 60 m with a loop equivalent to one of circular radius 6 m. Curves A, B and C are for half spaces with resistivities of 10, 100 and 1000 Ωm respectively. Airborne single loop TEM thus looks promising, especially for resistive terrains. However the success of airborne TEM would depend upon being able to buck out helicopter response. This has not been included in the calculations.

Lastly, a calculation for the 'worst case' likely to be encountered in ground TEM surveys has been done. The attenuation effects due to height increase with increasing loop height/loop radius ration, increasing ground conductivity, and early times.

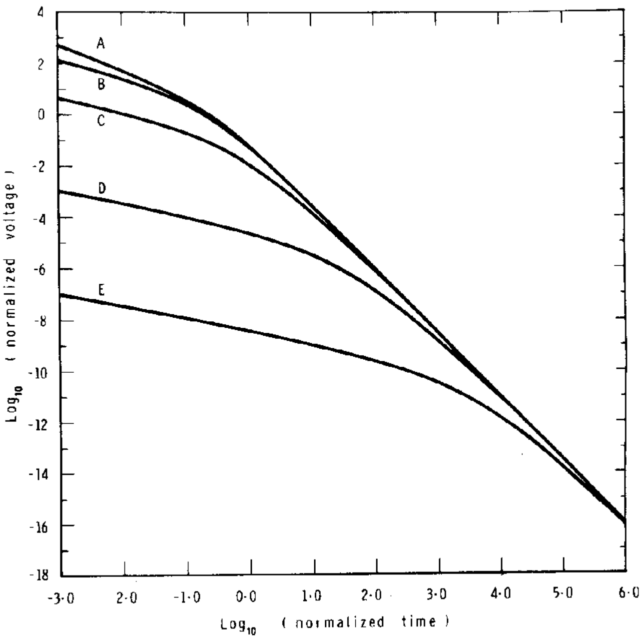

FIGURE 1
Logarithmic decay curves of normalised voltage for five values of H (loop height/loop ratio). A, H = 0; B, H = 0.1; C, H = 1; D, H = 10; E, H = 100.

†Specific cases of interest not covered by Fig. 1 can be obtained by contacting the author.

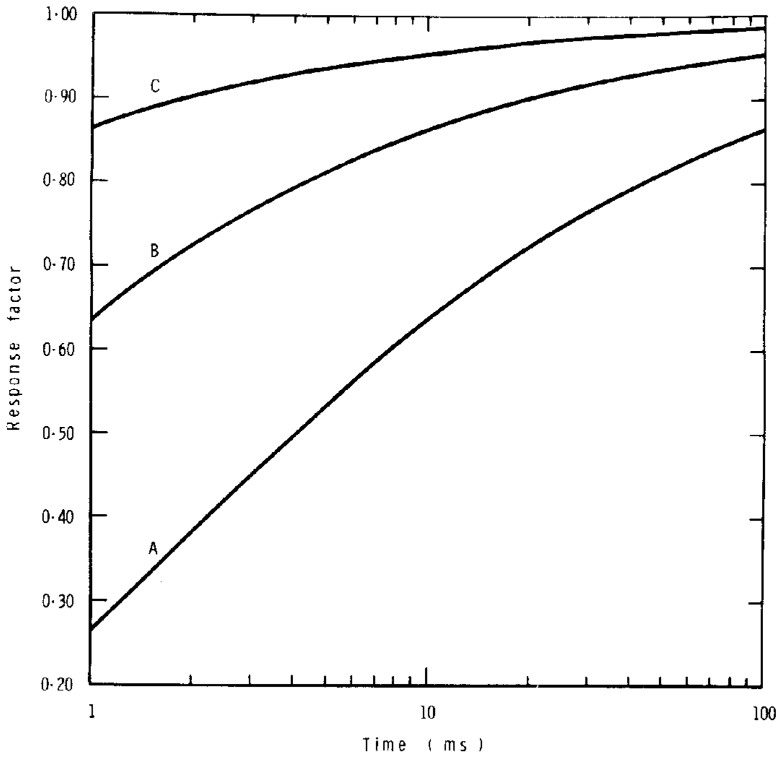

FIGURE 2
Altitude response factor for half-spaces with resistivities of 10(A), 100(B) and 1000(C) Ωm.

Thus for $\sigma = 1$ s/m, t = 0.4 ms, h = 2, and L = 25m, the response at h = 2 m is 80% of that of the h = 0 response; i.e. a correction factor of –20%. Using Lee's 'exact' first order approximation gives a correction of –23%. Using the simple approximation $125h\sqrt{\frac{\sigma\mu\pi}{t}}$ gives a correction of –25%. Thus for most ground surveys, the simple correction formula is valid. In the extreme case presented, the simple formula is only 25% in error.

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