Paper

Depth Estimation with PEM —A Cautionary Note

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The method advocated by Crone (1977) for determining the position of a "current axis" for a conducting ore body will give a positive result even if there is no ore body.

The depth, ℓ, to this axis in a uniform ground of conductivity σ and permeability μ_0 is given by:

$$\ell = \frac{8}{5} \cdot \frac{1}{\sqrt{\pi}} \sqrt{\frac{t}{\sigma \mu_0}}$$

In this expression t is time and consequently the result can be used to determine the conductivity profile of a nearly uniform ground.

1. Introduction

There are few methods of estimating the depth and position of a conductive body that are available to the field geophysicist. It is pleasing, then, to learn of the method advocated by Crone (1977) to do this task. In what follows it will be argued that this method should be treated with caution as it will tive a positive result over a uniform ground.

The method under discussion has been described by Crone (1977) as follows: "In this method a vertical detailed transmitting loop is placed over the weak anomaly for the purpose of selectively energising the conductor at depth. A short traverse with six to eight readings spaced twenty to twenty-five metres apart is carried out over the suspected coneuctor, perpendicular to its strike. Readings are taken at both horizontal and vertical receiving loop positions and the dip angles of the secondary field at all eight samples are plotted. If perpendicular lines are drawn from the dip angle then they should converge at the eddy current axis of that sample".

Figure 1 shows the construction used in the interpretation.

2. Theory

As the loops used in this field survey are quite small they can be approximated by a horizontal magnetic dipole. The electromagnetic fields about such a vertical loop resting on a uniform half space of conductivity σ and magnetic permeability μ_0 have been given by Morrison *et al* (1969), and Ward *et al* (1973).

Whence,

$$H_x = \frac{-Q}{x 4\pi} \int_0^\infty (1 + R(\lambda))\lambda\left[-J_1(\lambda x) + \lambda x J_0(\lambda x)\right] d\lambda \tag{1}$$

$$H_y = 0 \tag{2}$$

$$H_z = \frac{+Q}{4\pi} \int_0^\infty (R(\lambda) - 1)\lambda^2 J_1(\lambda x)\, d\lambda \tag{3}$$

$$R(\lambda) = (\sqrt{\lambda^2 + i\omega\sigma\mu_0} - \lambda)/(\sqrt{\lambda^2 + i\omega\sigma\mu_0} + \lambda) \tag{4}$$

In these equations Q is the dipole moment, i.e. $N_1 A_1 I$ where N_1 is the number of turns in the loop and A_1, the area of the loop and I the current flowing in the loop. Here displacement currents have been neglected and an e^{iwt} time dependence has been assumed.

H_x, H_y and H_z are the three components of the magnetic field. The axes chosen have their origin at the centre of the loop with the x axis directed out along the axis of the loop. This axis defines the traverse line. The z axis is directed vertical and the y axis is into the page. See Figure 1.

The spectra of the voltages induced in a small receiving loop ($\bar{V}_x$, $\bar{V}_z$) orientated vertically or horizontally for a step-like current flowing in the primary loop are given by:

$$\bar{V}_x = -A_2 N_2 H_x \tag{5}$$

$$\bar{V}_z = -A_2 N_2 H_z \tag{6}$$

Here we notice that the spectrum of a step function is $1/\omega$ and this cancels with the $i\omega$ arising from the time derivative of the magnetic field. Also A_2 is the area of the small receiving loop of N_2 turns.

The time domain voltages ($V_x(t)$, $V_z(t)$) therefore are:

$$V_x(t) = -\frac{Q}{x}\frac{N_2 A_2}{4\pi}\int_0^{\infty}\frac{2\lambda^2}{\sigma\mu_0}F(\lambda,t)\left[-J_1(\lambda x)+\lambda x J_0(\lambda x)\right]d\lambda \tag{7}$$

$$V_z(t) = +\frac{Q N_2 A_2}{4\pi}\int_0^{\infty}\frac{2\lambda^3}{\sigma\mu_0}F(\lambda,t)J_1(\lambda x)d\lambda \tag{8}$$

If t is time then for $t > 0$.

$$F(\lambda,t) = \frac{a}{\sqrt{\pi}}e^{-\lambda^2/a^2} - \lambda\,\mathrm{erfc}\left(\frac{\lambda}{a}\right) \tag{9}$$

where $a = \sqrt{\sigma\mu_0/t}$

(Lee and Lewis (1974)).

These integrals are readily evaluated by firstly using the power series expansion for the Bessel functions, Abromowitz and Stegun (1965, F.9.1.10) and secondly the results of Gradshteyn and Rhzshik (1965, No. 6.281).

Expanding the Bessel function as a power series and integrating term by term the resulting integrands of equations 7 and 8 yields:

$$V_x = \frac{-QA_2N_2}{x\sigma\mu_0}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^{2n+1}\frac{(-1)^n\,a^{2n+5}\,(2n+1)}{n!\,4\pi\,\sqrt{\pi}\,(2n+5)} \tag{10}$$

$$V_z = \frac{+QA_2N_2}{\sigma\mu_0}\sum_{n=0}^{\infty}\left(\frac{x}{2}\right)^{2n+1}\frac{(-1)^n\,a^{2n+6}\,\Gamma(n+5/2)}{n!\,(n+1)!\sqrt{\pi}\,(2n+6)\,4\pi} \tag{11}$$

The angle that fixes the "current axis" is seen from Figure 1 to be defined by

$$\tan\theta = -V_z/V_x \tag{12}$$

This expression simplifies for $ax \ll 1$ to be:

$$\tan\theta = \frac{5\sqrt{\pi}}{8}ax \tag{13}$$

This result shows that even for a uniform half space there is a "current axis" at a depth ℓ, where

$$\ell = \frac{8}{5}\cdot\frac{1}{\sqrt{\pi}}\cdot\sqrt{\frac{t}{\sigma\mu_0}} \tag{14}$$

This equation shows that the apparent depth is independent of station position.

3. Discussion

The above result does not show that the method will not yield the "current axis" of a steeply dipping conductor beneath the transmitter. It does show, however, that a positive result will be obtained even when there is no conductor present. For this reason the method of depth estimation should be treated with care.

Alternatively equation 14 can be used to estimate the conductivity of a reasonably uniform ground once ℓ is known. As noted above this quantity can be found from Figure 1.

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5. References

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