The Effect of Loop Height in Transient Electromagnetic Modelling or Prospecting

T. J. Lee

L. A. Richardson & Assoc. P/L.,
P.O. Box 217,
Gordon, N.S.W. 2072

The voltage induced in a horizontal loop over a uniform ground at height, h, has been calculated for the case where the loop is excited by a step current. If the loop has a radius b and the ground has a permeability of μ and conductivity then at time t :

$$\left(\frac{\partial V}{\partial h}\frac{100}{V}\right)\Big|_{h=o} \simeq 125\sqrt{\frac{\delta\mu\pi}{t}}$$

For $\sigma\mu b^2/(4t)$ in the range 10^{-4} to 1 the maximum error in using this formula is less than 44%. If is less than 0.1 the error is less than 5%.

It is suggested, then, that for small h,

$$V(h) \simeq V(o) + \frac{\partial V}{\partial h}\Big|_{h=o} h$$

and that the above formulae will give first order corrections for the voltage induced in the loop for changes in elevation of the loop.

Introduction

At the present time all the type curves for the interpretation of transient electromagnetic data assume that the traverses are carried out on flat terrain. Further, in the case of model studies carried out by using metallic models all the dimensions are very small. A consequence of this is that the actual field situation being modelled is for a loop several metres above the ground. There is a need, therefore, to be able to allow for the elevation changes of these loops.

The Calculation of Elevation Changes

If V is the voltage observed in a loop of radius b over a uniform half-space of conductivity δ and h is the height of the loop above the ground, then:

$$V(h,\sigma,b) = V(o,\sigma,b) + \sum_{n=1}^{\infty} \frac{\partial^n}{\partial h^n} V(h,\sigma,b)\Big|_{h=o} \frac{h^n}{n!} \tag{1}$$

Therefore for small elevations the percentage change per metre of height is approximately

$$\frac{(V(h,\sigma,b) - V(o,\sigma,b))\times 100}{h\,V(o,\sigma,b)} = \frac{\frac{\partial V(h,\sigma,b)\times 100}{\partial h}}{V(o,\sigma,b)} \tag{2}$$

From Lee and Lewis (1974) (equation 2 and 13), the secondary voltage E induced in the loop carrying a e^{iwt} current is:

$$\bar{E} = i\pi b^2\omega\mu I\int_0^{\infty} e^{-2\lambda h}\left[J_1(\lambda b)\right]^2\left(\frac{n_1-\lambda}{n_1+\lambda}\right)d\lambda$$

$$\text{where} \qquad n_1 = \sqrt{\lambda^2 + i\omega\mu\sigma} \tag{3}$$

Here μ is the permeability of the ground and is assumed to be equal to the permeability of free space, μo.

The voltage, $\bar{V}$, due to a step function wave form is then:

$$\bar{V} = \pi b^2\mu I\int_0^{\infty} e^{-2\lambda h}\left[J_1(\lambda b)\right]^2\left(\frac{n_1-\lambda}{n_1+\lambda}\right)d\lambda \tag{4}$$

Therefore

$$\frac{\partial \bar{V}}{\partial h}\Big|_{h=o} = -2\pi b^2\mu I\int_0^{\infty}\lambda\left[J_1(\lambda b)\right]^2\left(\frac{n_1-\lambda}{n_1+\lambda}\right)d\lambda \tag{5}$$

Writing x = λb and denoting the inverse Laplace transform by $\mathcal{L}^{-1}$ yields for $t > 0$.

$$\frac{\partial V}{\partial h}\Big|_{h=o} = \frac{4\pi b\mu I}{b}\int_0^{\infty}x^2\left[J_1(x)\right]^2\mathcal{L}^{-1}\left(\frac{1}{m+x}\right)dx \tag{6}$$

where $m_1 = \sqrt{x^2 + i\omega\mu\sigma b^2}$ and $\mathcal{L}^{-1}(\bar{V}) = V$

Therefore

$$\frac{\partial V}{\partial h}\Big|_{h=o} = \frac{4\pi\mu I}{b^2\sigma\mu\sqrt{\pi\tau}}\int_0^{\infty}x^2\left[J_1(x)\right]^2 e^{-\tau x^2}\left(1 - x\sqrt{\pi\tau}\,e^{\tau x^2}\,\mathrm{erfc}(x\sqrt{\tau})\right)dx \tag{7}$$

Here $\tau = t/\sigma\mu b^2$

The above expression can be evaluated by expanding the Bessel functions as a power series and then evaluating the resulting integrals by using the definition of the gamma function and the result given by Gradshteyn and Ryzhik (1965, p648, Number 6.281):

$$\int_0^{\infty} x^{m-1}\operatorname{erfc}(x)\,dx = \frac{\Gamma\left(\frac{m+1}{2}\right)}{m\sqrt{\pi}} \tag{8}$$

Therefore

$$\left.\frac{\partial v}{\partial h}\right|_{h=0} = \frac{4\pi\mu I}{\sigma\mu b^2}\sum_{m=0}^{\infty}\left\{\frac{(-1)^m(2m+2)!\,(2m+3)!!}{2^m m!\,(m+1)!\,(m+1)!\,(m+2)!\,(m+3)}\left(\frac{\sigma\mu b^2}{4t}\right)^{m+3}\right\} \tag{9}$$

From Lee and Lewis (1974, equation 11)

$$V = -\frac{2b\mu I\sqrt{\pi}}{t}\left(\frac{\sigma\mu b^2}{4t}\right)^{3/2}\sum_{m=0}^{\infty}\left\{\frac{(-1)^m(2m+2)!\left((\sigma\mu b^2)/4t\right)^m}{(m+1)!\,(m+2)!\,(2m+5)}\right\} \tag{10}$$

Therefore

$$\left.\left(\frac{\partial v}{\partial h}\,\frac{100}{v}\right)\right|_{h=0} = 25\sqrt{\frac{\sigma\mu\pi}{t}}\left\{\frac{\sum_{m=0}^{\infty}\frac{(-1)^m(2m+2)!(2m+3)!!\left((\sigma\mu b^2)/(4t)\right)^m}{2^m(m)!(m+1)!(m+1)!(m+2)!(m+3)}}{\sum_{m=0}^{\infty}\frac{(-1)^m(2m+2)!\left((\sigma\mu b^2)/(4t)\right)^m}{m!(m+1)!(m+2)!(2m+5)}}\right\} \tag{11}$$

If $\frac{\sigma\mu b^2}{4t}$ is small, then: $\left.\left(\frac{\partial v}{\partial h}\,\frac{100}{v}\right)\right|_{h=0} \simeq 125\sqrt{\frac{\sigma\mu\pi}{t}}$ (12)

Discussion

Figure 1 shows the results of comparing the exact formula with the approximate formula for $\frac{\sigma\mu b^2}{4t}$ in the range 10^{-4} to 1. As the figure shows the percentage error is very small. In the practical field case the mean height can be of the order of one metre due to bushes, topography etc., however, this figure shows that the resultant error is always less than 1%. So for this case it would not be necessary to apply the correction.

However, in scale model experiments when the modelling materials are metals then the correction is important because the scaled coils are at a much greater distance from the surface.

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References

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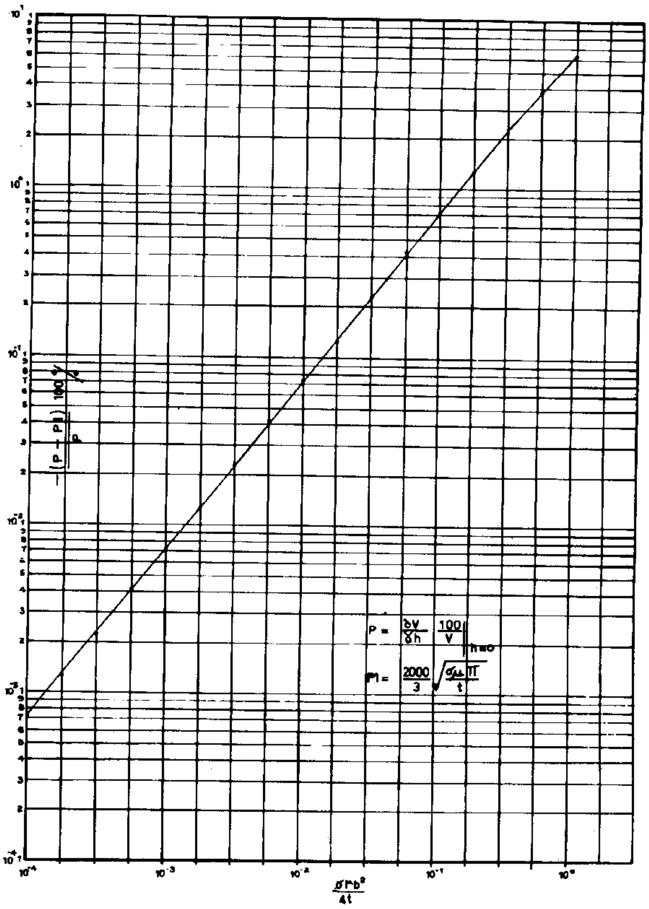

FIGURE 1
First Order Corrections for elevation changes for single loop TEM equipment.