Interpretation of magnetic gradient tensor for automatic locating a dipole source

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SUMMARY

In this paper, I propose the algorithm that the location of a magnetic dipole can be detected from the magnetic gradient tensor. I derive the location vector of a vertically magnetizated dipole from magnetic gradient tensor. Deficit of magnetic moment of magnetic dipole makes the induced location information incomplete. However if the observation of magnetic gradient tensor would be collected on one more points, the algorithm is able to point the location of magnetic dipole by clustering the solution of the proposed method. For example, I show that magnetic gradient tensor can be converted as the source location successively by picking common solution area in synthetic case of borehole observation.

Key words: magnetic gradient tensor, magnetic dipole

INTRODUCTION

Recently, the magnetic gradient survey has been utilized in many practical cases such as detecting UXO (Sanchez, et al, 2005) and archaeological application (von der Osten-Woldenburg, et al, 2006). The magnetic gradient survey has better advantages rather than conventional total magnetic intensity (TMI) survey because it has better resolution of shallow targets (Schmidt and Clark, 2006).

In order to interpret the magnetic gradient tensor data, I derive a closed-form solution that point out the source location from observation point. This method begins with the relationship between a magnetic dipole and the magnetic gradient tensor. This method has the same analogy of Rim and Li (2010) that dealt with gravity gradient tensor. And back to original, it is analogous to the methods developed by Morris et al (1995). In case of gravity gradient tensor, Pederson and Rasmussen (1990) derived several invariant relations between gravity gradient tensor, and Beike and Pedersen (2010) successively applied these invariants on interpretation of gravity gradient tensor. However their method required all components for calculating invariants because obtaining eigenvector included all components. In this work, I derive analytic relation between source and observing points using only three independent components rather than all.

DERIVATION OF THE RELATION BETWEEN THE SOURCE LOCATION AND MAGNENT GRADIENT TENSOR

The magnetic gradient tensor of a vertically magnetizated dipole is a tensor defined by the double gradient of a positioning vector as

$$\Gamma \equiv \begin{bmatrix} B_{xx} & B_{xy} & B_{xz} \\ B_{yx} & B_{yy} & B_{yz} \\ B_{zx} & B_{zy} & B_{zz} \end{bmatrix} = -\nabla\nabla M \hat{\mathbf{z}} \cdot \nabla \frac{1}{\left|\vec{r_q} - \vec{r_p}\right|} \tag{1}$$

where M is magnetic moment of dipole and $\hat{\mathbf{z}}$ is unit vector for vertical direction. $\vec{r_p} = (x_p, y_p, z_p)$ and $\vec{r_q} = (x_q, y_q, z_q)$ is the positioning vector of the observation point and source respectively (Blakely, 1996). The magnetic gradient tensor components are given in matrix form as

$$\Gamma = M\frac{3}{R^7}\begin{bmatrix} z(-R^2+5x^2) & 5xyz & x(-R^2+5z^2) \\ 5xyz & z(-R^2+5y^2) & y(-R^2+5z^2) \\ x(-R^2+5z^2) & y(-R^2+5z^2) & z(-3R^2+5z^2) \end{bmatrix} \tag{2}$$

For simplicity, I defined the positioning vector from observation point to source location $\mathbf{r} = (x, y, z) = (x_q - x_p, y_q - y_p, z_q - z_p)$ and its distance is $R = \sqrt{(x_q - x_p)^2 + (y_q - y_p)^2 + (z_q - z_p)^2}$. This tensor has five independent components as it is symmetric and traceless. The direction angles can be obtained through ratios of proper tensor components and additional parameter ε. ε can be derived from composition of tensor components as like Appendix A.

$$\begin{aligned} \theta_{xy} &= \tan^{-1}\left(\frac{x}{y}\right) = \tan^{-1}\left(\frac{B_{xz}}{B_{yz}}\right) \\ \theta_{yz} &= \tan^{-1}\left(\frac{y}{z}\right) = \tan^{-1}\left(\frac{2-3\varepsilon}{4-\varepsilon}\frac{B_{yz}}{B_{zz}}\right) \\ \theta_{zx} &= \tan^{-1}\left(\frac{z}{x}\right) = \tan^{-1}\left(\frac{4-\varepsilon}{2-3\varepsilon}\frac{B_{zz}}{B_{xz}}\right) \end{aligned} \tag{3}$$

In equation (3), θ_{xy}, θ_{yz}, and θ_{xz} are the angles from $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ to the source, counter-clockwise on the x-y, y-z, and z-x plane respectively. The schematic relation between positioning vector and its direction angles are shown as Fig. 1. Using direction angles and one of magnetic gradient tensor component, the positioning vector can be derived as

$$\mathbf{r} = \pm|\mathbf{r}|\varsigma(\cos\theta_{yz}\cos\theta_{xy}, \cos\theta_{yz}\sin\theta_{xy}, \sin\theta_{yz}\sin\theta_{xy}) \tag{4}$$

where

$$|\mathbf{r}| = \left\{ (3M)^2 \frac{(2-3\varepsilon)^2}{(1+\varepsilon)^3} \frac{1}{B_{zz}^2} \right\}^{1/8}$$

$$\varsigma = \left(\cos^2\theta_{xy} \cos^2\theta_{yz} + \sin^2\theta_{xy} \right)^{-1/2} .$$

However the positioning vector still has ambiguities because the magnetic moment is unknown. Therefore I propose automatic locating method with connecting magnetic gradient measurements.

LOCATING THE DIPOLE FROM MAGNETIC GRADINET TENSOR

During recovering positioning vector, the magnetic moment has still unknown. Therefore the exact location cannot be derived because of insufficient information of magnetic moment. In order to obtain the location of magnetic dipole, I proposed the method to find a vertically magnetized dipole without magnetic moment information.

1. set up a certain size window in given observation points (x_p, y_p, z_p) and tolerance T that the solution could be accepted within. And then searching points (x_k, y_k, z_k) are also settled down by discretising the interested area.

2. calculate positioning vectors without magnetic moment information at given observation points in the selected window,

$$\mathbf{r} \propto \varsigma(\cos\theta_{yz}\cos\theta_{xy}, \cos\theta_{yz}\sin\theta_{xy}, \sin\theta_{yz}\sin\theta_{xy})$$

where

$$\varsigma = \left(\cos^2\theta_{xy} \cos^2\theta_{yz} + \sin^2\theta_{xy} \right)^{-1/2} .$$

3. obtain directional cosine (c_1, c_2, c_3) at given observation points using direction angles.

$$(c_1, c_2, c_3) = \varsigma\left(\cos\theta_{xy}\cos\theta_{yz}, \sin\theta_{xy}\cos\theta_{yz}, \sin\theta_{xy}\sin\theta_{yz} \right)$$

4. if it is satisfied with equation (5) at every observation points (x_p, y_p, z_p) within the selected window, the point (x_k, y_k, z_k) could be a true source location.

$$\left(a_1c_1 + a_2c_2 + a_3c_3 \right)^2 - \left(a_1^2 + a_2^2 + a_3^2 - \tau^2 \right) \geq 0 \qquad (5)$$

where $(a_1, a_2, a_3) = (x_p - x_k, y_p - y_k, z_p - z_k)$.

5. repeat the procedure from 1 to 4 until the moving window covers all observation points.

The equation (5) comes from the condition that the line with directional cosine (c_1, c_2, c_3) and through the observation point (x_p, y_p, z_p) should be passed through within the sphere with center (x_k, y_k, z_k) and radius τ. If the solutions would be fallen within the clustered area, the area can be picked the true point source location.

SYNTHETIC CASE

I simulated magnetic gradient measurement of a vertically magnetizated dipole in borehole as shown Fig. 2. In case using magnetic gradient components B_{xz} , B_{yz} , and B_{zz} , the proposed method can converse solution in the vicinity of real source area.

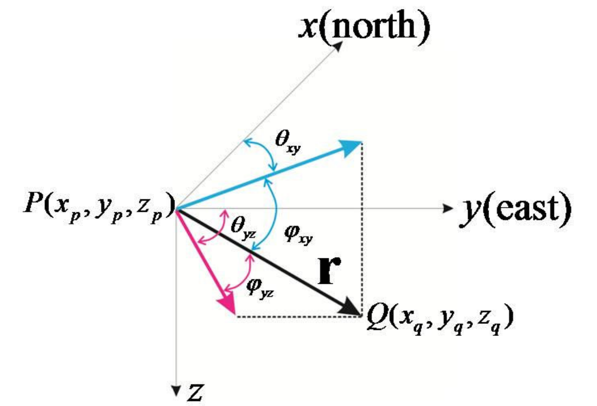

Figure 1. Schematic relation between $\mathbf{r}$, θ and φ . $\mathbf{r}$ is the positioning vector between observation point $P(x_p, y_p, z_p)$ and location of magnetic dipole $Q(x_q, y_q, z_q)$. θ_{xy} is the angle between x axis and projected positioning vector on $x-y$ plane and θ_{yz} is the angle between y axis and projected positioning vector on $y-z$ plane. φ_{xy} and φ_{yz} are the projection angles of positioning vector on the $x-y$ plane and $y-z$ plane, respectively.

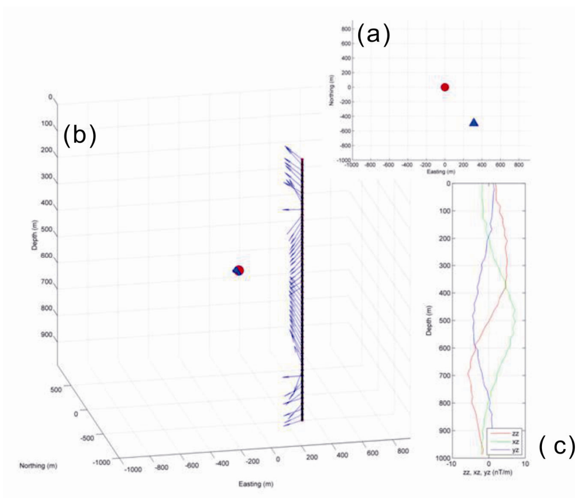

Figure. 2. Locating a magnetic dipole using magnetic gradient tensor. (a) In upper panel, the red circle and blue triangle represented the location of dipole source and observation borehole, respectively. (b) In main panel, the algorithm proposed in the text showed the perfect match between the solutions (blue dots) and real magnetic dipole location (red solid circle) even though the magnetic tensor had a white noise. The blue arrows which were calculated by directional cosines on the observation borehole pointed the location of magnetic dipole. (c) In right panel, B_{xz} , B_{yz} and B_{zz} are shown as three independent components of magnetic gradient in case calculated on the borehole observation points. Each component has 5% Gaussian noise.

CONCLUSIONS

The location of magnetic dipole was derived from magnetic gradient tensor. However it is impossible to induce the exact location vector of magnetic dipole completely because the magnetic moment of dipole is unknown. In order to point the location of magnetic dipole without magnetic moment I proposed the algorithm to cluster the solution area from one more observation points of magnetic gradient tensor. In synthetic case of borehole measurements, the proposed method successively pointed the location from magnetic gradient tensor. So far, the algorithm is applied on synthetic case, it could be extended to real data if the distance between observation and source is far enough to approximate a single dipole source.

APPENDIX A

Derivation of ε

Firstly, new variable Ω_z is defined by ratio of magnetic gradient tensor components as

$$\Omega_z = \frac{B_{xz}^2 + B_{yz}^2}{B_{zz}^2} \qquad \text{(A-1).}$$

Ω_z can be rewritten by replacing magnetic gradient tensor components as

$$\Omega_z = \frac{\varepsilon(4-\varepsilon)^2}{(2-3\varepsilon)^2} \qquad \text{(A-2).}$$

The equation (A-2) can be solved as

$$\begin{aligned}
\varepsilon_1 &= -\frac{a}{3} + \eta^{1/3} - \beta\eta^{-1/3} \\
\varepsilon_2 &= -\frac{a}{3} - \frac{1}{2}\left(\eta^{1/3} - \beta\eta^{-1/3}\right) - \frac{\sqrt{3}}{2} i\left(\eta^{1/3} + \beta\eta^{-1/3}\right) \qquad \text{(A-3)} \\
\varepsilon_3 &= -\frac{a}{3} - \frac{1}{2}\left(\eta^{1/3} - \beta\eta^{-1/3}\right) + \frac{\sqrt{3}}{2} i\left(\eta^{1/3} + \beta\eta^{-1/3}\right)
\end{aligned}$$

where

$$\beta = \frac{b}{3} - \frac{a^2}{9}$$

$$\eta = \frac{ab}{6} - \frac{c}{2} + \left(\beta^3 + \left(\frac{a^3}{27} - \frac{ab}{6} + \frac{c}{2}\right)^2\right)^{1/2} - \frac{a^3}{27}$$

$$a = -9\Omega_z - 8$$
$$b = 12\Omega_z + 16$$
$$c = -4\Omega_z$$

One of solution in equation (A-3) can be selected by condition that ε should be positive real.

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